UDC 517.15

Recurrence Relations for Polynomials Orthonormal on Sobolev, Generated by Laguerre Polynomials

R. M. Gadzhimirzaev

Ramis M. Gadzhimirzaev, https://orcid.org/0000-0002-6686-881X, Dagestan Scientific Center, Russian Academy of Sciences, 45, Gadjieva Str., Makhachkala, Russia, 367025, ramis3004@gmail.com

In this paper we consider the system of polynomials $l_{r,n}^{\alpha}(x)$ (r — natural number, $n = 0, 1, \ldots$), orthonormal with respect to the Sobolev inner product (Sobolev orthonormal polynomials) of the following type $\langle f, g\rangle = \sum_{\nu=0}^{r-1} f^{(\nu)}(0)g^{(\nu)}(0) + \int_0^{\infty} f^{(r)}(t)g^{(r)}(t)\rho(t)\,dt$ and generated by the classical orthonormal Laguerre polynomials. Recurrence relations are obtained for the system of Sobolev orthonormal polynomials, which can be used for studying various properties of these polynomials and calculate their values for any x and n. Moreover, we consider the system of the Laguerre functions $\mu_n^{\alpha}(x) = \sqrt{\rho(x)}l_n^{\alpha}(x)$, which generates a system of functions $\mu_{r,n}^{\alpha}(x)$ orthonormal with respect to the inner product of the following form $\langle \mu_{r,n}^{\alpha}, \mu_{r,k}^{\alpha}\rangle = \sum_{\nu=0}^{r-1}(\mu_{r,n}^{\alpha}(x))^{(\nu)}|_{x=0}(\mu_{r,k}^{\alpha}(x))^{(\nu)}|_{x=0} + \int_0^{\infty}(\mu_{r,n}^{\alpha}(x))^{(r)}(\mu_{r,k}^{\alpha}(x))^{(r)}\,dx$. For the generated system of functions $\mu_{r,n}^{\alpha}(x)$, recurrence relations for $\alpha = 0$ are also obtained.

Key words: Laguerre polynomials, Sobolev-type inner product, Sobolev orthonormal polynomials, Laguerre functions.

DOI: 10.18500/1816-9791-2018-18-1-17-24

References

1. Sharapudinov I. I., Gadzhieva Z. D., Gadzhimirzaev R. M. Systems of functions orthogonal with respect to scalar products of Sobolev type with discrete masses generated by classical orthogonal systems. *Daghestan Electronic Mathematical Reports*, 2016, iss. 6, pp. 31–60 (in Russian).
2. Sharapudinov I. I., Magomed-Kasumov M. G. Cauchy problem solution representation by Fourier series of polynomials, orthogonal on Sobolev, generated by Laguerre polynomials. *Differential Equations*, 2018, vol. 54, no. 1, pp. 51–68.
3. Marcellán F., Xu Y. On Sobolev orthogonal polynomials. *Expositiones Math.*, 2015, vol. 33, iss. 3, pp. 308–352. DOI: 10.1016/j.exmath.2014.10.002.
4. Pérez T. E., Piñar M. A., Xu Y. Weighted Sobolev orthogonal polynomials on the unit ball. *J. Approx. Theory*, 2013, vol. 171, pp. 84–104. DOI: 10.1016/j.jat.2013.03.004.
5. Delgado A. M., Fernández L., Lubinsky D. S., Pérez T. E., Piñar M. A. Sobolev orthogonal polynomials on the unit ball via outward normal derivatives. *J. Math. Anal. and Appl.*, 2016, vol. 440, iss. 2, pp. 716–740. DOI: 10.1016/j.jmaa.2016.03.041.
6. Fernández L., Marcellán F., Pérez T. E., Piñar M. A., Xu Y. Sobolev orthogonal polynomials on product domains. *J. Comput. and Appl. Math.*, 2015, vol. 284, pp. 202–215. DOI: 10.1016/j.cam.2014.09.015.
7. Lopez G., Marcellán F., Van Assche W. Relative asymptotics for polynomials orthogonal with respect to a discrete Sobolev inner product. *Constr. Approx.*, 1995, vol. 11, iss. 1, pp. 107–137. DOI: 10.1007/BF01294341.

8. Sharapudinov I. I., Sharapudinov T. I. Polynomials orthogonal in the Sobolev sense, generated by Chebyshev polynomials orthogornal on a mesh. *Russian Math. (Iz. VUZ)*, 2017, vol. 61, no. 8, 59–70. DOI: 10.3103/S1066369X17080072.
9. Gadzhimirzaev R. M. The Fourier series of the Meixner polynomials orthogonal with respect to the Sobolev-type inner product. *Izv. Saratov Univ. (N. S.), Ser. Math. Mech. Inform.*, 2016, vol. 16, iss. 4, pp. 388–395 (in Russian). DOI: 10.18500/1816-9791-2016-16-4-388-395.
10. Szego G. *Orthogonal Polynomials*. AMS Colloq. Publ., 1939, vol. 23. 440 p.

Cite this article as:
Gadzhimirzaev R. M. Recurrence Relations for Polynomials Orthonormal on Sobolev, Generated by Laguerre Polynomials. *Izv. Saratov Univ. (N. S.), Ser. Math. Mech. Inform.*, 2018, vol. 18, iss. 1, pp. 17–24 (in Russian). DOI: 10.18500/1816-9791-2018-18-1-17-24.