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Stability of Periodic Billiard Trajectories in Triangle

A. N. Kirillov, R. V. Alkin

Aleksandr N. Kirillov, https://orcid.org/0000-0002-3356-1846, Institute of Applied Mathematical Research of the Karelian Research of the Russian Academy of Science, 11, Pushkinskaya Str., Petrozavodsk, Republic of Karelia, Russia, 185910; Petrozavodsk State University, 33, Lenin Str., Petrozavodsk, Republic of Karelia, Russia, 185910, kirillov@krc.karelia.ru
Ruslan V. Alkin, https://orcid.org/0000-0002-8257-4520, Petrozavodsk State Univercity, 33, Lenina Str., Petrozavodsk, Republic of Karelia, Russia, 185910, ruslan5.ru@yandex.ru

The problem of stability of periodic billiard trajectories in triangles is considered. The notion of stability means the preservation of a period and qualitative structure of a trajectory (its combinatorial type) for sufficiently small variations of a triangle. The geometric, algebraic and fan unfoldings are introduced for stable trajectories description. The new method of fan coding, using these unfoldings, is proposed. This method permits to simplify the stability analysis. The notion of code equivalence and combinatorial type of a trajectory is proposed for trajectories classification. The rigorous definition of stable periodic trajectory in a triangle is formulated. The necessary and sufficient conditions of a fan code stability are obtained (Theorem 1). In order to simplify the stable periodic trajectories classification the notion of pattern, is introduced which permits us to generate the stable codes (Theorem 2). The method of stable periodic trajectories construction is proposed (Theorem 3). The introduced notions are illustrated by several examples, particularly for trajectories in obtuse triangles. The possibility of application of the developed instrument to obtuse triangles offers opportunities of its using to solve the problem of the existence of periodic billiard trajectories in obtuse triangles. A new notion of periodic billiard trajectory conditional stability, relating to some special variations, is introduced.

Key words: mathematical billiard, coding of trajectories, stability, pattern, fan code.

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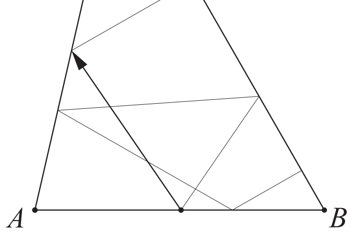

Fig. 1. 12-linked trajectory

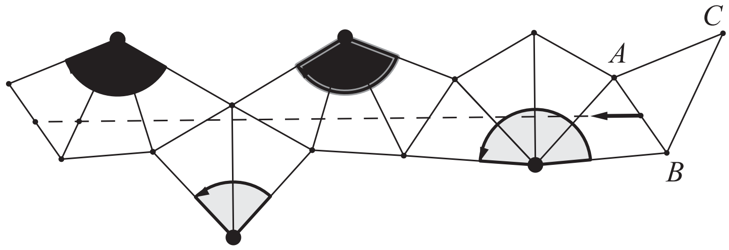

Fig. 2. The fan unfolding of a 12-linked trajectory. Turns in different directions are marked by different fillings

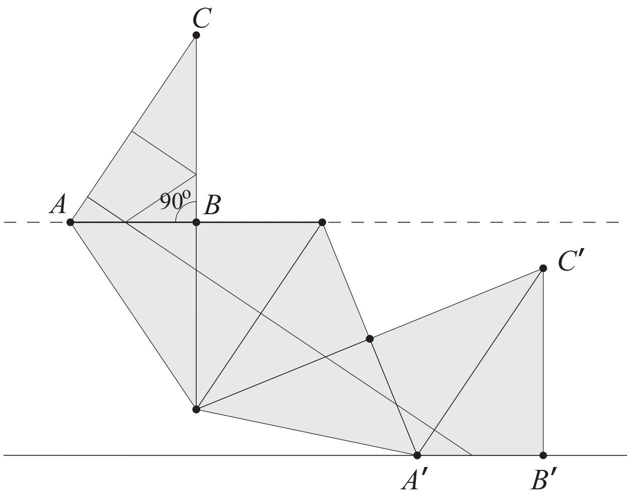

Fig. 5. Original rectangle triangle, $\alpha = 56.08°$, $\beta = 90.00°$

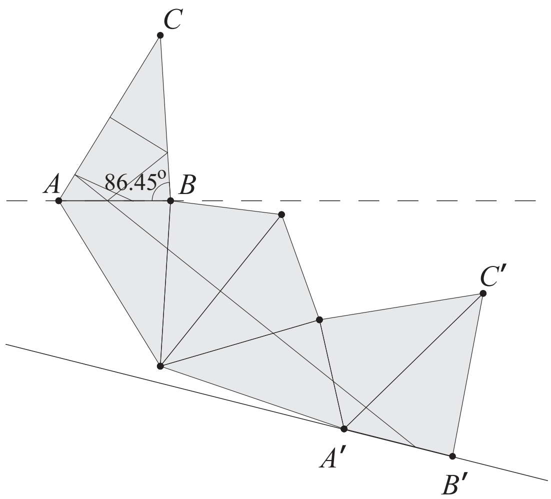

Fig. 6. The perturbed triangle, $\alpha = 58.45°$ $\beta = 86.45°$

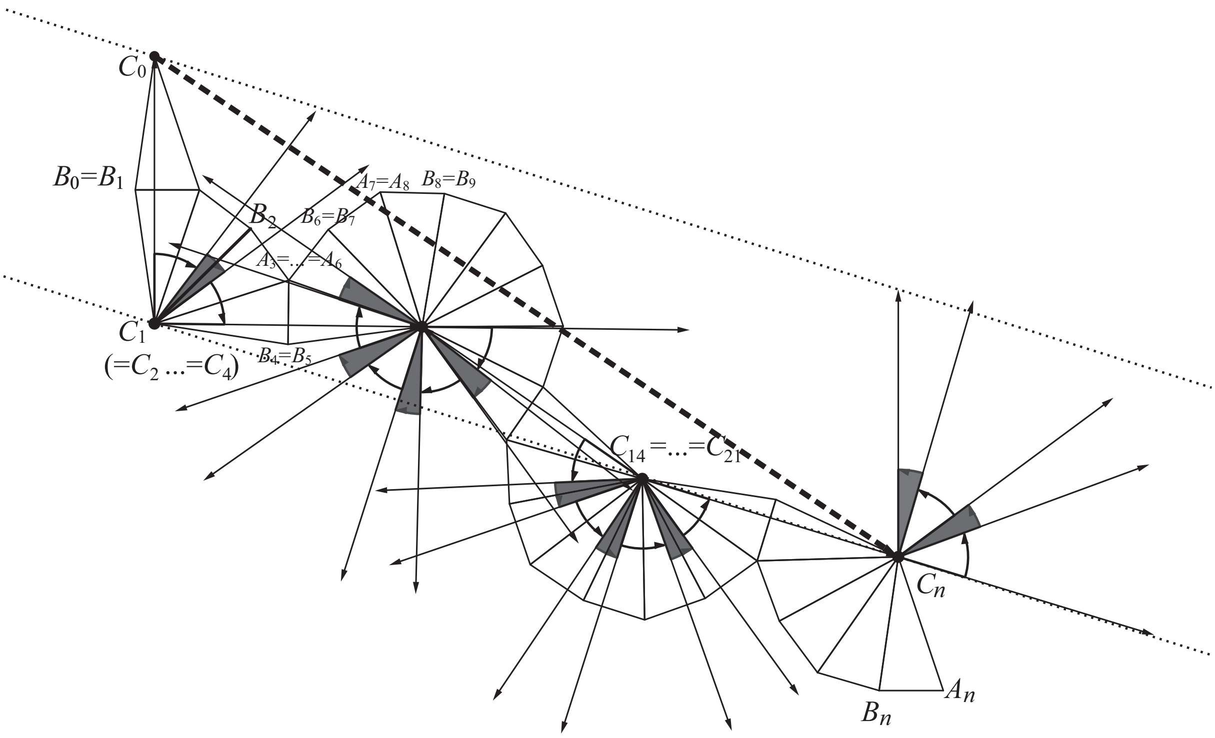

Fig. 7. Rotations of the vector $\vec{C_1C_0}$ to the angles $\widehat{ca}$ (light angles) and to the angles $\widehat{cb}$ (dark angles). $\vec{C_0C_n}$ is the direction vector of the trajectory (dotted line)

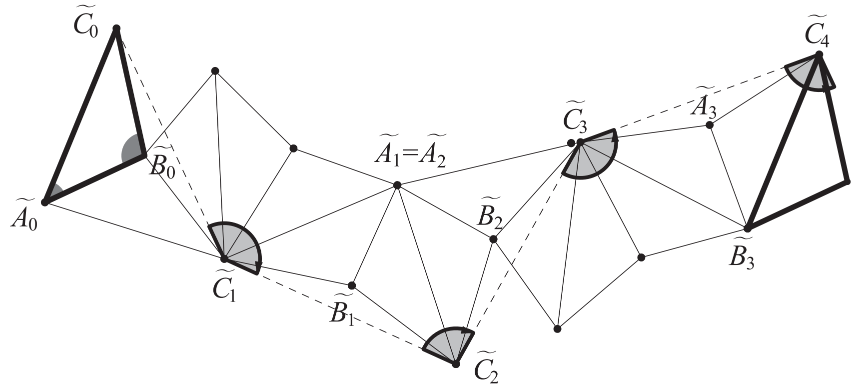

Fig. 8. A fan unfolding for the code $\{+4, +1, -4, -1\}$ and the triangle $\triangle(42^\circ, 102^\circ)$

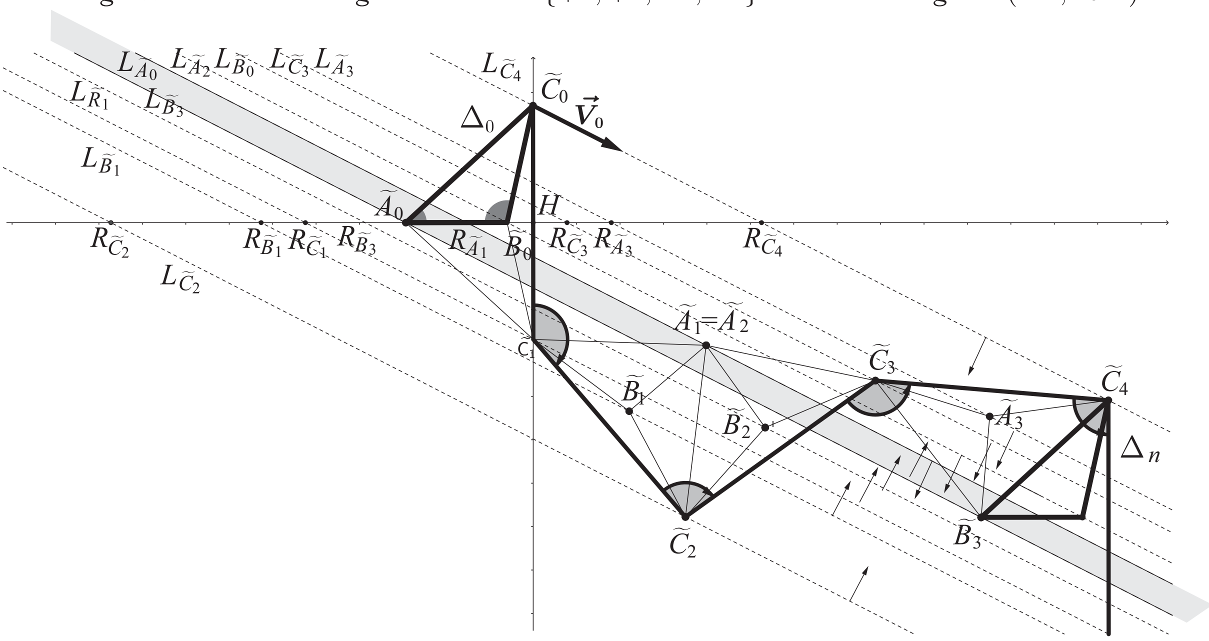

Fig. 9. The direction vector $\vec{V_0}$ of the trajectory and the admissible corridor for the code $\{+4, +1, -4, -1\}$ for the triangle $\triangle(42^\circ, 102^\circ)$. Bold lines indicate a broken line connecting the centers of fans, as well as $\triangle_0$ and $\triangle_n$. The arrows indicate the half-planes containing the corridor

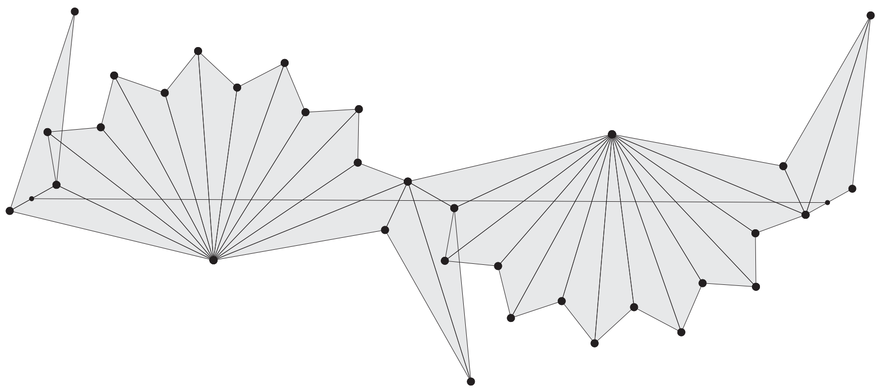

Fig. 10. An example of implementing the pattern $+0 + 1 - 0 - 1$ with the code $+12 + 1 - 12 - 1$

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