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Asymptotic Formulae for Weight Numbers of the Sturm – Liouville Boundary Problem on a Star-shaped Graph

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In this article the Sturm – Liouville boundary value problem on the graph Γ of a special structure is considered. The graph Γ has m edges, joined at one common vertex, and m vertices of degree 1. The boundary value problem is set by the Sturm – Liouville differential expression with real-valued potentials, the Dirichlet boundary conditions, and the standard matching conditions. This problem has a countable set of eigenvalues. We consider the so-called weight numbers, being the residues of the diagonal elements of the Weyl matrix in the eigenvalues. These elements are meromorphic functions with simple poles which can be only the eigenvalues. We note that the considered weight numbers generalize the weight numbers of Sturm – Liouville operators on a finite interval, equal to the reciprocals to the squared norms of eigenfunctions. These numbers together with the eigenvalues play a role of spectral data for unique reconstruction of operators.We obtain asymptotic formulae for the weight numbers using the contour integration, and in the case of the asymptotically close eigenvalues the formulae are got for the sums. The formulae can be used for the analysis of inverse spectral problems on the graphs.

Key words: Sturm – Liouville boundary problem, asymptotic formulae, weight numbers, star-shaped graph.

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