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On the Representation of Functions by Absolutely Convergent Series by $\mathscr{H}$-system

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The paper deals with the representation of absolutely convergent series of functions in spaces of homogeneous type. The definition of a system of Haar type ($\mathscr{H}$-system) associated to a dyadic family on a space of homogeneous type X is given in the Introduction. It is proved that for almost everywhere (a.e.) finite and measurable on a set X function f there exists an absolutely convergent series by the system $\mathscr{H}$, which converges to f a.e. on X. From this theorem, in particular, it follows that if $\mathscr{H} = \{h_n\}$ is a generalized Haar system generated by a bounded sequence $\{p_k\}$, then for any a.e. finite on $[0,1]$ and measurable function f there exists an absolutely convergent series in the system $\{h_n\}$, which converges a.e. to $f(x)$. It is also proved, that if X is a bounded set, then one can change the values of an a.e. finite and measurable function on a set of arbitrary small measure such that the Fourier series of the obtained function with respect to system $\mathscr{H}$ will converge uniformly. The paper results are obtained using the methods of metrical functions theory.

Key words: Haar type system, dyadic family, absolute convergence, uniform convergence.

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