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Bending of a Sandwich Beam by Local Loads in the Temperature Field

E. I. Starovoitov, D. V. Leonenko

Eduard I. Starovoitov, https://orcid.org/0000-0002-2550-5377, Belarusian State University of Transport, 246653, Belarus, Gomel, Kirova Str., 34, edstar0@yandex.by

Denis V. Leonenko, https://orcid.org/0000-0001-8003-9279, Belarusian State University of Transport, 246653, Belarus, Gomel, Kirova Str., 34, leoden@tut.by

Deformation of sandwich beam in a temperature field under the action of uniformly distributed and sinusoidal local loads is considered. An analytical view of the loads was set by using functions of Heaviside. To describe kinematic properties of an asymmetric through thickness of sandwich beam we have accepted the hypotheses of a broken line as follows: Bernoulli's hypothesis is true in the thin bearing layers; Timoshenko's hypothesis is true in the compressible through thickness filler with a linear approximation of displacements through the layer thickness. The kinematic conditions of simply supported faces of the beam on the immovable in space rigid bases are presumed on the boundary. The filler's work is taken into account in the tangential direction. Temperature variations were calculated by the formula obtained from averaging thermophysical properties of the materials of the layers through the beam thickness. Stress and strain are related by relations of the deformation theory of plasticity. By the variational method a system of differential equilibrium equations has been derived. The solution of the boundary value problem of thermo-elastoplasticity is reduced to the search for four functions, namely: deflections and lengthwise displacements of the medial surfaces of the bearing layers. An analytical solution has been derived by the method of elastic solutions. In the case of repeated alternating loading solution using Moskvitin theorem received. Numerical analysis of solutions is performed for a continuous, locally distributed and repeated alternating loads. The graphs of stresses and displacements in sandwich beam under the isothermal and thermal-force loads are given.

Key words: local uniformly distributed and sinusoidal loads, sandwich elastic-plastic beam, compressible filler, temperature field.

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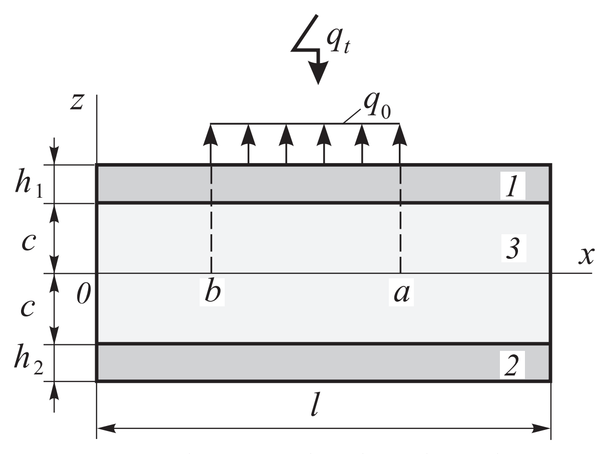

Fig. 1. Design diagram of a three-layer beam with a compressible filler: *1, 2* — load-carrying layers; *3* — filler

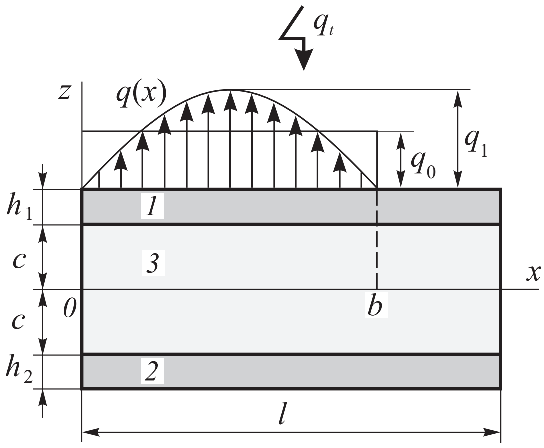

Fig. 2. Design diagram of a three-layer beam with a local sinusoidal loads

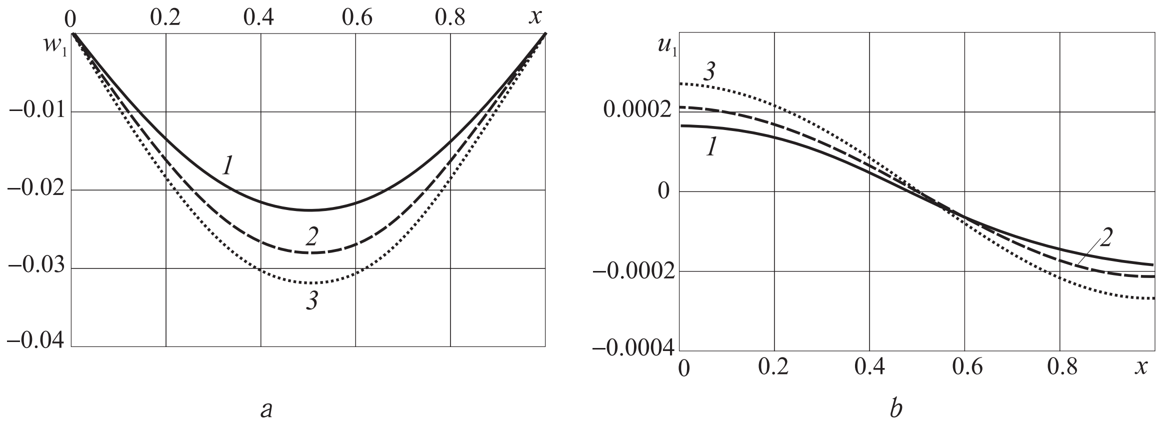

Fig. 3. Deflection w_1 (*a*) and longitudinal displacements u_1 (*b*) in the first layer along the axis of the statically equivalent distributed loads: *1* — rectangular; *2* — sinusoidal (both isothermal); *3* — sinusoidal temperature and force

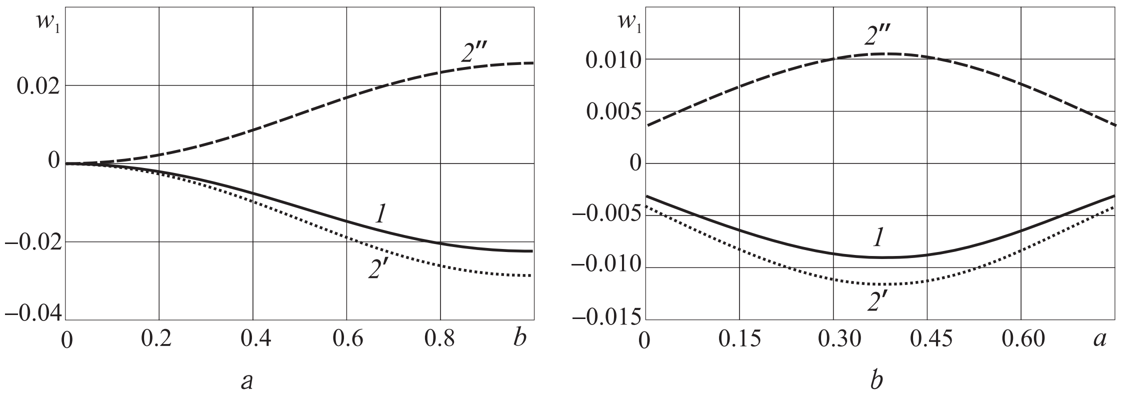

Fig. 4. Deflection w_1 in the middle of beams vs. coordinates b (*a*) and a (*b*) of ends of the interval of locally distributed load: elastic (*1*), thermoelastoplastic in loading from the natural state (*2′*), and thermoplastic in repeated sign-variable loading (*2″*)

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