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Approximate Theory of a Laminated Anisotropic Plate Vibrations

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The multi-layered plate vibration is investigated. A two-dimensional asymptotic model of the second order accuracy with respect to the small thickness parameter is proposed with account for the transverse shear and the normal fibre extension. The model is appropriate for a monoclinic plate described by 13 elastic moduli which is heterogeneous in the thickness direction. In particular, the model can be applied to a multi-layered plate consisting of orthotropic layers of arbitrary orientation. In this case the elastic moduli are piece-wise constant functions. The elastic and inertia properties of plate are assumed to be constant in the tangential directions. The main achievement of this work is derivation of the equivalent constant coefficients of 2D system of partial differential equations of the second order accuracy. In the first approximation these coefficients can be found based on the Kirchhoff – Love hypotheses on the straight normal, while a more complex asymptotic algorithm is used for second approximation. For a multi-layered plate the influence of transverse shear with alternating hard and soft layers is discussed. More attention is given to a plate which is infinite in the tangential directions. The solution is shown to be essentially simplified since no boundary condition is needed and the solution can be expressed in terms of functions which are harmonic in the tangential directions. For this solution the error of 2D model is estimated by comparison with the numerical solution of the three-dimensional problem of elasticity theory, since for harmonic case it is reduced to one-dimensional equations in the thickness direction. Free and forced bending vibration and long-length bending wave propagation are investigated under harmonic approximation. In general case two different natural frequencies are shown to correspond to a fixed bending mode. The dependence of wave velocity on the wave propagation direction is found out.

Key words: anisotropic multi-layered plate, 2D model of the second order accuracy, bending vibrations and waves in a plate.

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Table 1

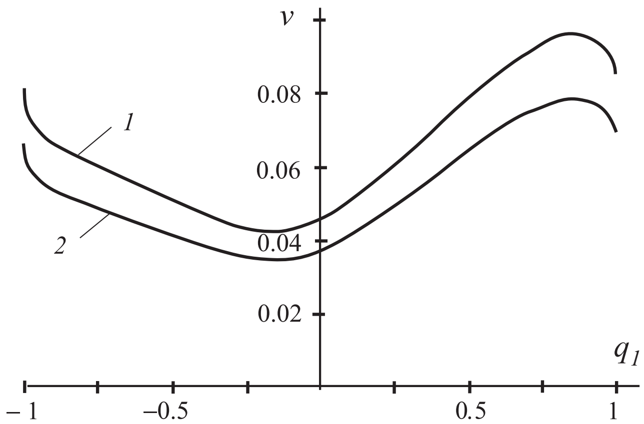
Elastic moduli of layers

N	E_{11}	E_{22}	E_{33}	$E_{12}=E_{13}=E_{23}$	$E_{44}=E_{55}=E_{66}$
1,5	12.0	2.0	2.0	0.59	0.69
2,4	1.1	1.1	1.1	0.33	0.38
3	1.0	11.0	1.0	0.30	0.35

Table 2

Dependence of the frequency parameter on the ratio of the elastic moduli η

1	2	3	4	5	6	7
η	λ_e	λ_0	$\varepsilon(\%)$	λ_a	$\varepsilon(\%)$	δ_s
1	0.002211	0.002229	0.9	0.002212	0.0	0.0079
0.1	0.001985	0.002073	4.5	0.001986	0.1	0.044
0.01	0.001482	0.002058	38.5	0.001459	1.6	0.41
0.001	0.000511	0.002056	303.0	0.000405	20.9	4.07



Functions $v(q_1)$ for symmetric (*1*) and for asymmetric (*2*) plates

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References

1. Ambartsumyan S. A. *Obshchaya teoriya anizotropnyh obolochek* [General theory of anisotropic shells]. Moscow, Nauka, 1974. 448 p. (in Russian).
2. Rodionova V. A., Titaev B. A., Chernykh K. F. *Prikladnaya teoriya anizotropnyh plastin i obolochek* [Applied theory of anisotropic plates and shells]. St. Petersburg, St. Petersburg Univ. Publ., 1996. 280 p. (in Russian).
3. Agolovyan L. A. *Asimptoticheskaya teoriya anizotropnyh plastin i obolochek* [Asymptotic theory of anisotropic plates and shells]. Moscow, Nauka, 1997. 414 p. (in Russian).
4. Reddy J. N. *Mechanics of laminated composite plates and shells*. CRC Press, 2004. 306 p.
5. Vetukov Y., Kuzin A., Krommer M. Asymptotic splitting of the three-dimensional problem of elasticity for non-homogeneous piezoelectric plates. *Int. J. of Solids and Structures*, 2011, vol. 48, iss. 1, pp. 12–23. DOI: https://doi.org/10.1016/j.ijsolstr.2010.09.001
6. Schnieder P., Kienzler R. An algorithm for the automatization of pseudo reductions of PDE systems arising from the uniform-approximation technique. *Shell-like structures. Non-classical theories and applications*. Berlin, Springer, 2011, pp. 377–390.

7. Tovstik P. E., Tovstik T. P. Generalized Timoshenko – Reissner models for beams and plates, strongly heterogeneous in the thickness direction. *ZAMM*, 2017, vol. 97, iss. 3, pp. 296–308. DOI: https://doi.org/10.1002/zamm.201600052
8. Tovstik P. E., Tovstik T. P. An elastic plate bending equation of second-order accuracy. *Acta Mech.*, 2017, vol. 228, iss. 10, pp. 3403–3419. DOI: https://doi.org/10.1007/s00707-017-1880-x
9. Morozov N. F., Tovstik P. E., Tovstik T. P. Generalized Timoshenko – Reissner model for a multi-layer plate. *Mechanics of Solids*, 2016, vol. 51, no. 5, pp. 527–537. DOI: https://doi.org/10.3103/S0025654416050034
10. Tovstik P. E., Tovstik T. P. Two-dimensional model of a plate made of an anisotropic inhomogeneous material. *Mechanics of Solids*, 2017, vol. 52, no. 2, pp. 144–154. DOI: https://doi.org/10.3103/S0025654417020042
11. Tovstik P. E., Tovstik T. P., Naumova N. V. Long-wave vibrations and waves in anisotropic beam. *Vestnik SPbSU. Mathematics. Mechanics. Astronomy*, 2017, vol. 4 (62), iss. 2, pp. 323–335. DOI: https://doi.org/10.21638/11701/spbu01.2017.216
12. Morozov N. F., Belyaev A. K, Tovstik P. E., Tovstik T. P. Two-dimensional equations of the second order accuracy for a multi-layered plate with orthotropic layers. *Doklady Physics*, 2018, vol. 63, no. 11, pp. 471–475.
13. Schnieider P., Kienzler R. A Reissner-type plate theory for monoclinic material derived by extending the uniform-approximation technique by orthogonal tensor decompositions of nth-order gradients. *Meccanica*, 2017, vol. 52, iss. 9, pp. 2143–2167. DOI: https://doi.org/10.1007/s11012-016-0573-1

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