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## On the Unsymmetrical Buckling of Shallow Spherical Shells under Internal Pressure

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This work is devoted to the numerical study of unsymmetrical buckling of shallow spherical shells and annular plates with varying mechanical characteristics subjected to internal pressure. We suppose that the edge of the shell is clamped but moving freely in the shell's plane. For the annular plate a roller support is considered for the inner edge of the plate, i.e. the edge that can slide along the figure axes without changing the slope. The unsymmetric part of the solution is sought in terms of multiples of the harmonics of the angular coordinate. A numerical method is employed to obtain the lowest load value, which leads to the appearance of waves in the circumferential direction. The effect of material inhomogeneity on the buckling load is examined. It is shown that if the elasticity modulus decreases away from the center of a plate, the critical pressure for unsymmetric buckling is sufficiently lower than for a plate with constant mechanical properties.

**Key words:** shallow spherical shell, annular plate, unsymmetrical buckling, inhomogeneity.

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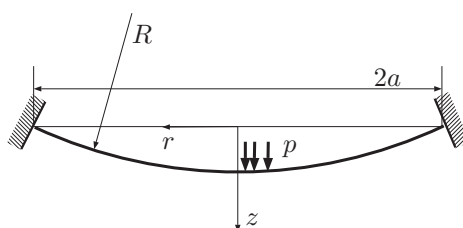


Fig. 1. Spherical shallow shell

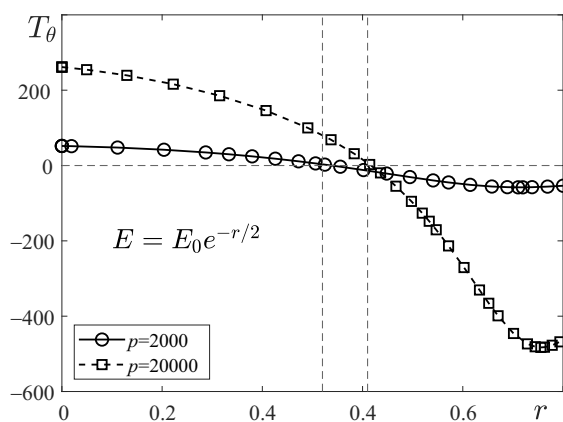


Fig. 2. Dimensionless circumferential stress resultant for a heterogeneous panel loaded with internal pressure ( $A = 2.5$ ,  $E = E_0 e^{-0.5r}$ )



*Table 1*  
Critical load for a homogeneous shallow spherical panel  
( $A = 0$  corresponds to a circular plate)

|          |       |       |       |       |       |       |
|----------|-------|-------|-------|-------|-------|-------|
| $A$      | 0     | 0.5   | 1     | 2     | 3     | 4     |
| $p_{cr}$ | 64522 | 65763 | 67394 | 70361 | 73633 | 77110 |
| $n$      | 14    | 14    | 14    | 15    | 16    | 16    |

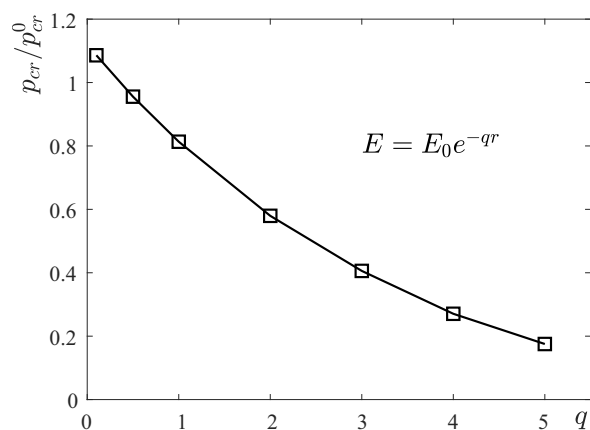


Fig. 3. The change of the critical load when the degree of heterogeneity of the panel  $q$  changes.  $A = 2.5$ ,  $p_0$  — critical load for a uniform circular plate

*Table 2*  
Critical load for a homogeneous annular plate ( $\delta = 0$  corresponds to a circular plate)

|                   |    |      |      |      |
|-------------------|----|------|------|------|
| $\delta$          | 0  | 0.05 | 0.1  | 0.15 |
| $p_{cr}/p_{cr}^0$ | 1  | 1.03 | 1.08 | 1.15 |
| $n$               | 14 | 13   | 12   | 12   |

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