Рассмотрена расширенная модель гибридных автоматов для динамических систем, где наряду с дискретной управляющей подсистемой и объектами управления с сосредоточенными по пространству параметрами имеются объекты управления с распределенными по пространству параметрами (линейные и стационарные с точки зрения теории автоматического управления). Показана возможность программной реализации расширенной модели гибридных автоматов на встроенных вычислительных системах.

Ключевые слова: гибридные автоматы, гибридные динамические системы, встроенные вычислительные системы.

Multiple Hedging on Energy Market

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The article is devoted to the calculation of the dynamic hedge ratio based on three different types of volatility models, among which S-BEKK-GARCH model takes into account cross-sectional dependence. The hedging strategy is built for eight stock-futures pairs on energy market in Russia.

Keywords: multivariate volatility models, spatial specifications, dynamic hedge ratio, energy market.

INTRODUCTION

Energy market can significantly influence economy on different levels. Therefore, hedging of energy price risk has become an important issue for households, firms and policy makers. Due to recent increase of energy price volatility a wide range of papers devoted to volatility estimation and hedging strategy building have been published (see in [1, 2]).

The original hedging strategies assumed that conditional volatility is unchanged in time [3], i.e. they were static. Later new approaches have been proposed including dynamic hedging strategies, as well as cross-hedging. Futures contracts are one of the most widely used financial assets for hedging due to such features as fixed term of expiration, low transaction cost, high liquidity and low margin requirement.

Dynamic hedging strategies, based on a time-dependent optimal hedge ratio, allow managing a portfolio of different assets considering cross-correlation of spot and futures contracts. To calculate the optimal hedge ratio, the first step is to investigate the time-varying volatility transmission among the assets using one of the parametric models, for example, VAR-GARCH [4], BEKK-GARCH [5], Markov switching GARCH [6].

In [4] VAR-GARCH and CCC-GARCH (constant conditional correlations) models are used to define volatility spillovers between oil and stock prices and the authors come to the conclusion that a better understanding of such links is crucial for portfolio management. The results also show that for all the pairs, consisted of oil and stock from some economic sector, short position of financial sector stock is the most effective strategy to hedge oil price risks.

Time-varying conditional correlations and hedge ratios of Goldman Sach's Energy index pared with S&P 500 are examined in [7]. The authors implement a BEKK-GARCH specification for conditional variances and include US dollar index as an independent variable in mean equation. According to their empirical results, energy index show better performance as a hedge instrument for equities during extreme downturns.

The linkage of crude oil market (proxy of oil price is the West Texas Intermediate crude oil price) and stock markets of the G-7 countries are studied in [5]. They combine a bivariate BEKK-GARCH model with wavelet approach in order to analyze volatility spillovers and compute hedging ratios and optimal weights that are varying across scales.

Moreover, Wang and Liu [8] investigate volatility spillovers and dynamic correlations between crude oil and stock markets using BEKK-, CCC- and DCC-GARCH models. They focus on the dynamic relationships between markets in countries that have been divided into oil-exporting and oil-importing. As a result, in this hedging strategy crude oil risk can be better hedged by indices of oil-exporting countries. It happens because of the higher level of sensitivity to geopolitical events compared to oil-importing countries.

In [9] an attempt is made to investigate the time-varying correlations using the multivariate DECO-FIEGARCH approach. As for hedging, their results show that gold could be better instrument than oil while reducing stock price risk.

There is a large number of different approaches that increase hedging efficiency. Ghoddusi and Emamzadehfard [10] apart from estimation of contagion effects consider the maturity of the futures contracts and build three different hedge estimation methods, based on OLS, error correction model and GARCH. Among them OLS hedging strategy shows the best performance on short and long horizons.

For example, the researchers [2] implement quantile hedge ratio and compare it with minimum variance for three energy-related commodities: crude oil, heating oil and natural gas. In a case of long hedging horizons, they firstly use wavelet analysis to decompose the daily return series and find that on horizon of four weeks the both ratios for crude oil and heating oil converge.

The aim of this paper is to introduce spatial effects, i. e. cross-sectional dependencies, in building the hedging strategy. The idea to take into account cross-sectional dependencies in finance came from spatial econometrics and was introduced by [11]. Their model is based on the assumption that a lot of the cross-sectional dependencies between the stock returns can be captured by three different types of dependence: a general dependence, dependence within industrial branches, and dependence based on geographic locations.

For instance, authors [12] focus on the modelling of dependencies among the world financial markets and ability to forecast them. They propose a simple FDI-based measure of financial distance. The use of this measure in model significantly reduces the mean squared error in returns prediction and helps to capture the dependencies in world financial markets.

There is already some evidence that spatial models are good in optimal hedging ratio calculation, forecasting, modeling the effects of contamination and volatility spillovers in comparison with other multivariate models [13–15].

In this paper, we build hedging strategy following methodology of time-variant hedge ratio computation described in [16]. In contrast to previous studies, we rely on such multivariate volatility model as BEKK-GARCH with spatial effects and use generalized orthogonal (GO-GARCH) and dynamic conditional correlations (DCC) GARCH models as a benchmark, (for more details, see [17,18]). The comparison was made using hedging effectiveness proposed by [3]. We use data from Russian financial market, investigating eight companies from energy sector during 2011–2018.

The rest of the paper is organized as follows: section 1 gives a short description of

multivariate GARCH models used in the article. Section 2 describes methodology of hedging strategy building. Section 3 provides information about the dataset. Section 4 reports estimation results and section 4 concludes.

1. MULTIVARIATE GARCH MODELS

Let x_t, $x_t = (x_{1t}, x_{2t}, \ldots, x_{nt})'$ be a portfolio consisted of n assets at time moment t. x_t is represented as a sum of its mathematical expectation $\mathrm{E}(x_t | \mathscr{F}_t)$, conditional on all available at $t-1$ information, and innovations y_t,

$$x_t = \mathrm{E}(x_t | \mathscr{F}_{t-1}) + y_t, \quad t = 1, \ldots, T. \tag{1}$$

Innovations y_t conditional on volatility H_t are distributed normally with zero mean,

$$y_t | H_t \sim N(0, H_t). \tag{2}$$

For the purposes of hedging strategy building, we focus our attention on volatility matrix H_t. Its specifications used in the paper are described below.

1.1. S-BEKK model

The spatial specification of the multivariate model of generalized autoregressive conditional heteroscedasticity (spatial BEKK) allows to take into account both temporal and spatial effects in the dynamics of volatility [17]. Such effects are modelled using weight matrix that is given exogenously and can be defined either as a binary matrix or as a function of the economic distances [19].

The variance-covariance matrix H_t in spatial BEKK model has the following structure:

$$H_t = C'C + A'_{t-1}y_{t-1}y'_{t-1}A_{t-1} + B'_{t-1}H_{t-1}B_{t-1}, \tag{3}$$

where coefficient matrices A, B, C are defined in (4)–(7) and contain spatial component in dynamic weight matrices W_t.

$$A_{t-1} = \mathrm{diag}(a_0) + \mathrm{diag}(a_1)W_{t-1}, \tag{4}$$
$$B_{t-1} = \mathrm{diag}(b_0) + \mathrm{diag}(b_1)W_{t-1}, \tag{5}$$
$$C'C = D^{-1}\mathrm{diag}(d_0)(D')^{-1}, \tag{6}$$

where constant matrix D is computing as:

$$D = I_n - \mathrm{diag}(d_1)\overline{W}, \tag{7}$$

where $\overline{W}$ — mean value of dynamic weight matrix W_t.

Although parameter matrices have parameters only on main diagonals, such parametrization implies that parameter matrices are complete, owing to the presence of weight matrices. As a result, the model (3)–(7) provides plausible volatility dynamics together with reduced number of parameters, which is linear in n. In contrast, in original BEKK model [20] complete parameter matrices are obtained by at least $O(n^2)$ parameters.

The dynamic weight matrix W_t contains information about economic distance between assets and each element of matrix in time t, w_{ijt} is computed as follows,

$$w_{ijt} = \left(1 - \left(\frac{d_{ijt}}{b}\right)^2\right)^2, \text{if } j \text{ is one of the neighbours for } i. \tag{8}$$

In this matrix element d_{ijt} is the economic distance between assets (see section 3 for more details).

1.2. DCC-GARCH

The DCC model, proposed by [21], is defined as follows,

$$H_t = D_t R_t D_t, D_t = \text{diag}\left(\sqrt{h_t}\right), \tag{9}$$

$$h_t = \omega + Ay_{t-1} \odot y_{t-1} + Bh_{t-1}, \tag{10}$$

$$R_t = (\text{diag}(Q_t))^{-\frac{1}{2}} Q_t (\text{diag}(Q_t))^{-\frac{1}{2}}, \tag{11}$$

$$Q_t = (1 - \alpha - \beta)\bar{Q} + \alpha\varepsilon_{t-1}\varepsilon'_{t-1} + \beta Q_{t-1}, \tag{12}$$

A, B — diagonal matrices of parameters; D_t — diagonal matrix of conditional standard deviations of y_t; $\bar{Q}_t$ — unconditional variance matrix; R_t — conditional correlation matrix with unities on the main diagonal.

1.3. GO-GARCH

The GO-GARCH model is a special case of the BEKK model [22]. In this model the volatility matrix is parametrized as follows,

$$H_t = XV_t X', \tag{13}$$

where V_t — diagonal $n \times n$ matrix with univariate GARCH processes on the main diagonal; X — $n \times n$ matrix based on singular value decomposition, which is not time depended (see [22] for details).

2. DYNAMIC HEDGE RATIO

In this paper we implement multiple hedging approach instead of traditional one, where only two assets — spot and futures — are in the portfolio. Following [12], we compute the portfolio return y_p for the investor, who holds n spot positions, which are hedged with futures contracts, as

$$y_p = \omega' y_s - \beta' y_f, \tag{14}$$

where y_s — stock returns, y_f — futures returns, ω — $n \times 1$ vector of weights in unhedged portfolio, β — $n \times 1$ vector of hedge ratios.

The optimal vector of hedge ratios β^* in a case of minimum-variance approach can be obtained through

$$\frac{d\text{Var}(y_p)}{d\beta} = \left(\beta' H^f\right)' - \left(\omega' H^{sf}\right)' = 0, \tag{15}$$

where H^{sf} and H^f are variance-covariance matrices for the spot-futures pairs and futures respectively.

Therefore, the optimal vector of hedge ratios is:

$$\beta_t^* = \left(H_t^f\right)^{-1} H_t^{sf} \omega. \tag{16}$$

Without loss of generality we take unit vector for ω.

In the framework of the study we compare hedging strategies by hedging efficiency calculated as financial results of the portfolio (the sum of logarithmic returns) on forecast subsample and Sharp ratio.

Sharp ratio SR is obtained as

$$SR = \frac{y_p - y_n}{\mathrm{Var}(y_p)}, \tag{17}$$

where y_n — risk free ratio.

3. DATA DESCRIPTION

Our sample covers the data of eight companies from energy sector in Russia. The list of companies can be seen in Table 1. All the data are obtained from the Finam website (https://www.finam.ru/). Descriptive statistics of spot assets' returns can be found in Appendix, Table A1.

The close price was used as the daily price for both spot assets and futures. The dates range from June 20, 2011 until July 12, 2018 excluding public holidays, therefore the full sample contains 1681 observations.

Table 1

Tickers of spot assets

Company	Ticker	Company	Ticker
FSK YeES	FEES	Rosneft	ROSN
Gazprom	GAZP	RusHydro	HYDR
Lukoil	LKOH	Surgutneftegas	SNGS
Novatek	NVTK	Tatneft	TATN

The economic distances for weight matrices are calculated as a difference in trade volume (see example of weight matrix in Appendix, Table A2). Using a difference in trading volume as a proxy is caused by the fact that it is an endogenous variable and correlates with different measures of volatility [23]. [24] also point out that volatility of trading volume contains information about the intensivity of trading deals and incorporates the price effects of market activity arising because of speculators' or hedgers' strategies.

Econometric calculations were carried out on Amazon Elastic Compute Cloud service with RStudio Amazon Machine Image (AMI) installed (https://aws.amazon.com/). The AMI contains R version 3.3.1 running on Ubuntu 16.04 LTS (the AMI was developed by Louis Aslett, www.louisaslett.com/RStudio_AMI/).

4. EMPIRICAL RESULTS

The empirical results of this work include the evaluation of three multivariate GARCH models, namely DCC, GO-GARCH and S-BEKK; building three different hedging strategy based on them; comparison of the strategies obtained by hedging efficiency and Sharp ratio for the hedged portfolio.

Optimal minimum-variance hedge ratios are calculated according to (16) for the predicted values of volatility. As a forecast subsample we use the last one-third of the sample which include 560 observations.

The average, maximum and minimum values of the optimal HR are presented in Appendix, Table A3. The average values for S-BEKK model vary from 0.161 to 1.949. It should be noted that only for model with spatial effects the minimum value of HR is negative in the case of NVTK, FEES and HYDR. For others the values of optimal HR change from 0.303 to 1.493.

The hedging efficiency measures are presented in Table 2. The unconditional variance of portfolio is slightly different for the DCC and GO-GARCH models.

Table 2

Results on forecast subsample

Model	HE	SR	Var
S-BEKK	41.657	1.588	21.066
DCC	27.679	1.193	16.326
GO-GARCH	22.647	0.881	16.405

Note. HE — hedging efficiency, SR — Sharp ratio, Var — portfolio variance.

Hedging, based on the model with spatial effects (S-BEKK), allows to reduce portfolio risk to 21% from an initial value of 68% for non-hedged portfolio.

The S-BEKK model turns out to be the most effective and allows to achieve the greatest financial result — 42% and shows the highest value of Sharp Ratio — 1.588 (considering the risk free ratio — 8.2% [1]).

CONCLUSION

In this study we incorporate spatial dependencies between assets into hedging strategy. To calculate the economic distance we apply the difference between assets' trade volumes as a measure of economic proximity. The obtained hedge ratios are time-variant and calculated for such volatility models as DCC, GO-GARCH and S-BEKK GARCH. Therefore, the consideration of spatial effects in the S-BEKK model allows to increase the hedging efficiency to 42% and decrease risk to 21%.

The current research can be continued in several directions. First, portfolio can be supplemented by assets in other countries. Secondly, the financial indicator which is used for weight matrices computation can be changed to achieve better results in the estimation of volatility.

We believe that the results obtained in the paper could provide important implications for portfolio and risk management practitioners.

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Appendix

Table A1

Descriptive statistics of spot assets' return

Statistic	Mean	St. Dev.	Min	Q(25)	Median	Q(75)	Max
LKOH	0.055	1.564	−10.280	−0.820	0.048	0.911	9.240
GAZP	−0.019	1.641	−14.959	−0.927	−0.083	0.907	7.620
NVTK	0.058	1.945	−17.038	−0.981	0.024	1.054	9.363
ROSN	0.032	1.681	−8.054	−0.945	0.000	1.004	7.059
SNGS	0.003	1.785	−10.911	−0.935	0.000	0.926	8.750
TATN	0.086	2.165	−9.244	−1.163	0.052	1.299	11.730
FEES	−0.041	2.806	−24.655	−1.321	−0.050	1.211	15.551
HYDR	−0.040	2.140	−9.932	−1.178	−0.058	1.068	12.467

Note. Q(25) and Q(75) — 25%th and 75%th quantiles respectively. The rest designations are self-explanatory.

Table A2

Mean value of weight matrix, x100

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	0.00	7.25	7.22	7.23	7.25	7.23	7.19	7.19	7.20	7.20	6.72	7.19	6.95	6.95	6.86	0.38
2	7.25	0.00	7.22	7.23	7.25	7.24	7.19	7.20	7.20	7.21	6.71	7.19	6.94	6.94	6.86	0.38
3	7.24	7.24	0.00	7.21	7.25	7.23	7.19	7.17	7.19	7.17	6.73	7.17	6.94	6.96	6.87	0.43
4	7.25	7.25	7.21	0.00	7.24	7.24	7.18	7.19	7.19	7.18	6.72	7.20	6.93	6.95	6.86	0.40
5	7.25	7.26	7.23	7.24	0.00	7.24	7.19	7.18	7.19	7.20	6.70	7.19	6.94	6.94	6.86	0.39
6	7.25	7.26	7.23	7.24	7.24	0.00	7.16	7.18	7.18	7.19	6.73	7.18	6.94	6.98	6.85	0.39
7	7.23	7.23	7.20	7.22	7.22	7.20	0.00	7.19	7.19	7.18	6.72	7.18	6.92	6.97	6.91	0.43
8	7.23	7.23	7.19	7.20	7.21	7.20	7.20	0.00	7.20	7.19	6.74	7.20	6.98	6.96	6.90	0.38
9	7.22	7.21	7.18	7.19	7.19	7.19	7.17	7.17	0.00	7.24	6.76	7.25	6.99	6.98	6.88	0.37
10	7.21	7.23	7.18	7.19	7.21	7.20	7.18	7.18	7.23	0.00	6.73	7.25	6.96	6.98	6.88	0.39
11	6.92	6.85	6.99	6.94	6.84	6.90	6.86	6.88	6.89	6.87	0.00	6.92	6.91	6.87	7.02	3.36
12	7.21	7.21	7.17	7.20	7.19	7.19	7.16	7.17	7.25	7.28	6.74	0.00	6.96	7.00	6.89	0.40
13	7.06	7.03	7.05	7.03	7.10	7.05	7.02	7.02	7.07	7.06	6.72	7.08	0.00	7.20	6.92	1.59
14	7.06	7.04	7.05	7.04	7.03	7.14	7.11	7.04	7.10	7.08	6.81	7.14	6.99	0.00	6.97	1.40
15	6.96	6.96	7.00	6.97	6.96	6.97	7.12	7.03	7.04	6.99	7.01	7.02	6.96	6.92	0.00	2.10
16	4.17	4.11	5.12	4.77	4.49	4.91	5.45	7.35	4.45	4.06	15.1	4.53	11.5	10.0	10.0	0.00

Note. (1)–(8) and (9)–(16) are spot and futures tickers respectively as listed in Table 1.

Table A3

Statistics for optimal hedge ratio

Model	S-BEKK			DCC			GO-GARCH		
Ticker	max	min	mean	max	min	mean	max	min	mean
LKOH	2.423	0.548	1.544	1.060	0.841	0.870	0.865	0.303	0.852
GAZP	2.645	0.250	1.799	0.978	0.874	0.876	0.892	0.679	0.692
NVTK	1.078	−0.940	0.300	1.214	1.113	1.210	1.104	1.052	1.102
ROSN	2.482	0.199	0.961	0.882	0.691	0.705	1.219	1.103	1.110
SNGS	0.906	−0.667	0.161	0.527	0.366	0.400	0.620	0.552	0.558
TATN	2.678	0.155	1.286	1.493	1.178	1.231	1.188	1.171	1.172
FEES	1.479	−0.392	0.731	1.040	0.703	0.724	0.974	0.810	0.822
HYDR	2.470	−0.278	0.949	0.883	0.870	0.874	1.048	0.822	0.861

Note. Max, min and mean correspond to maximum, minimum and mean values of optimal hedge ratio.