A Couple Contact Loading at the Unilateral Contact of Beams

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The contact problem for the structure consisting of two beams is considered. The beams have the different lengths and the different variable thicknesses. One end of the shorter beam is clamped coinciding with the hinged end of the longer beam. The other ends of the beams are free. The given loading is applied to the longer beam. The beams undergo the weak joint bending with the unilateral (receding) contact. There is no friction between the beams. The bending of each beam is described by Bernoulli – Euler model. The contact problem is to find the contact loading, i.e. the forces of interaction of beams. This problem has a number of well-known characteristic features. Some of them inhere in the contact problems for the beam structures on the whole. The others inhere in the structures containing the beam that cannot be in the equilibrium for the arbitrary loading. Besides, this problem has the novel peculiarity consisting in the appearance of the concentrated couple in the contact loading. The non-negativity of the contact loading, as the necessary condition of the unilateral contact, is not spoiled because the concentrated couple is at the end of the beams and its "negative part" is located outside the beams and does not belong to the contact loading. The mathematical formulation of the contact problem is propounded. The uniqueness of the solution of this problem is proved. The analytical solution is constructed in some special cases. The relation between the problem under consideration and the well-known contact problem for two cantilever beams is established.

Keywords: Bernoulli – Euler beam, contact problem, unilateral contact, contact loading, concentrated couple, uniqueness of solution, analytical solution.

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References

1. Ponomaryov S. D., Biederman V. L., Likharev K. K., Makushin V. M., Malinin N. N., Feodosiev V. I. *Raschety na prochnost v mashinostroenii* [Stress Calculation in Mechanical Engineering]. Vol. 1. Moscow, Mashgiz, 1956. 884 p. (in Russian).
2. Feodosyev V. I. *Selected Problems and Questions in Strength of Materials*. Moscow, Mir, 1977. 432 p. (Rus. ed.: Moscow, Nauka, 1973. 400 p.)
3. Parhilovskii I. G. *Avtomobilnye listovye ressory* [Automotive Leaf Springs]. Moscow, Mashinostroenie, 1978. 232 p. (in Russian).
4. Osipenko M. A., Nyashin Yu. I., Rudakov R. N. A contact problem in the theory of leaf spring bending. *Int. J. Solids Struct.*, 2003, no. 40, iss. 12, pp. 3129–3136. DOI: https://doi.org/10.1016/S0020-7683(03)00112-4
5. Osipenko M. A. The contact problem for bending of a two-leaf spring with variable thicknesses of leaves. *Tomsk State University Journal of Mathematics and Mechanics*, 2014, no. 1(27), pp. 90–94 (in Russian).
6. Johnson K. L. *Contact Mechanics*. Cambridge, Cambridge Univ. Press, 1985. 452 p. (Rus. ed.: Moscow, Mir, 1989. 510 p.)
7. Kuznetsov S. A. *Mehanika kontaktnogo vzaimodeystvija* [Contact Mechanics]. Kazan, Kazan Univ., 2014. 72 p. (in Russian).

8. Osipenko M. A., Nyashin Yu. I., Kasatkin A. A. Osipenko M. A., Nyashin Yu. I., Kasatkin A. A. Singularities of contact problems for systems of strings and beams with weakly restrained elements. *PNRPU Mechanics Bulletin*, 2015, no. 1, pp. 121–129 (in Russian). DOI: https://doi.org/10.15593/perm.mech/2015.1.08
9. Aleksandrov V. M. Some Contact Problems for the Beams, Plates and Shells. *Inzhenerny Zhurnal*, 1965, vol. 5, no. 4, pp. 782–785 (in Russian).
10. Grigoluk E. I., Tolkachuov V. M. *Kontaktnye zadachi teorii plastin i obolochek* [The Contact Problems for Plates and Shells]. Moscow, Mashinostroenie, 1980. 415 p. (in Russian).
11. Li H., Dempsey J. P. Unbonded Contact of Finite Timoshenko Beam on Elastic Layer. *Journal of Engineering Mechanics*. 1988, July, vol. 114, no. 8, pp. 1265–1284. DOI: https://doi.org/10.1061/(ASCE)0733-9399(1988)114:8(1265)
12. Kravtchuk A. S. *Variatsionnye i kvazivariatsionnye neravenstva v mekhanike* [Variational and quasi-variational inequalities in mechanics]. Moscow, MGAPI, 1997. 340 p. (in Russian).
13. Osipenko M. A., Nyashin Yu. I. A Certain Approach to Solving of Some One-Dimensional Contact Problems. *Izv. Saratov Univ. (N. S.), Ser. Math. Mech. Inform.*, 2011, vol. 11, iss. 1, pp. 77–84 (in Russian).
14. Shi M., Srisupattarawanit T., Schiefer F., Ostermeyer G.-P. On the Wellbore Contact of Drill Strings in a Finite Element Model. *Proc. Appl. Math. Mech.*, 2013, no. 13, pp. 109–110. DOI: https://doi.org/10.1002/pamm.201310050
15. Kim J. H., Ahn Y. J., Jang Y. H., Barber J. R. Contact problems involving beams *Int. J. Solids Struct.*, 2014, no. 51, pp. 4435–4439. DOI: https://doi.org/10.1016/j.ijsolstr.2014.09.013
16. Vatulyan A. O., Vasilev L. V. Determination of Attaching Parameters of Inhomogeneous Beams in the Presence of Damping. *Izv. Saratov Univ. (N. S.), Ser. Math. Mech. Inform.*, 2016, vol. 16, iss. 4, pp. 449–456 (in Russian). DOI: https://doi.org/10.18500/1816-9791-2016-16-4-449-456
17. Starovoitov E. I., Leonenko D. V. Bending of a Sandwich Beam by Local Loads in the Temperature Field. *Izv. Saratov Univ. (N. S.), Ser. Math. Mech. Inform.*, 2018, vol. 18, iss. 1, pp. 69–83. DOI: https://doi.org/10.18500/1816-9791-2018-18-1-69-83
18. Aleksandrov V. M., Chebakov M. I. *Vvedeniye v mehaniku kontaktnyh vzaimodeistvii* [Introduction to the Contact Mechanics]. Moscow, Rostov-on-Don, LLC “ZVVR”, 2007. 114 p. (in Russian).
19. Rabotnov Yu. N. *Mehanika deformiruemogo tverdogo tela* [Mechanics of Deformable Solids]. Moscow, Nauka, 1988. 711 p. (in Russian).

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