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Multiple Completeness of the Root Functions of the Pencils of Differential Operators with Constant Coefficients and Splitting Boundary Conditions

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In the space of square summable functions on the main segment [0,1], the class of polynomial pencils of ordinary differential operators of the n-th order is considered. The coefficients of the differential expression are assumed to be constants. The boundary conditions are assumed to be splitting and two-point at the ends 0 and 1 (l of boundary conditions is taken only at the point 0, and the remaining $n - l$ is taken at the point 1). The differential expression and the boundary forms are assumed to be homogeneous, that is, they contain only main parts. It is supposed that roots of the characteristic equation of the pencils of this class are simple, non-zero and lie on two rays emanating from the origin in quantities k and $n - k$. Sufficient conditions of m-fold system completeness of root functions of the pencils of this class in the space of square integrable functions on the main segment (with a possible finite defect) are formulated. The multiplicity m completeness depends on the relations of the parameters n, l and k. In this case, it is assumed that some completely concrete determinants differ from zero. These determinants are constructed from the coefficients of the boundary forms of the pencil and the roots of the characteristic equation. An upper bound on a possible finite defect is given.

Keywords: pencil of ordinary differential operators, polynomial pencil of differential operators, homogeneous differential expression, homogeneous boundary forms, multiple completeness, root functions, eigen- and associated functions, derived chains, splitting boundary conditions.

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References

1. Naymark M. A. *Linear Differential Operators*. Moscow, Nauka, 1969. 526 p. (in Russian).
2. Keldysh M. V. On the completeness of the eigenfunctions of some classes of non-selfadjoint linear operators. *Russian Math. Surveys*, 1971, vol. 26, iss. 4, pp. 15–44. DOI: https://doi.org/10.1070/RM1971v026n04ABEH003985
3. Rykhlov V. S. On completeness of the root functions of polynomial pencils of ordinary differential operators with constant coefficients. *Taurida Journal of Computer Science Theory and Mathematics*, 2015, no. 1(26), pp. 69–86 (in Russian).
4. Keldysh M. V. On eigenvalues and eigenfunctions of certain classes of non-selfadjoint equations. *Dokl. Akad. Nauk SSSR*, 1951, vol. 77, no. 1, pp. 11–14 (in Russian).
5. Khromov A. P. *Finite-dimensional perturbations of Volterra operators*. Diss. Dr. Sci. (Phys. and Math.). Novosibirsk, 1973. 242 p. (in Russian).
6. Shkalikov A. A. The completeness of eigenfunctions and associated functions of an ordinary differential operator with irregular-separated boundary conditions. *Funct. Anal. Its Appl.*, 1976, vol. 10, iss. 4, pp. 305–316. DOI: https://doi.org/10.1007/BF01076030

7. Freiling G. Zur Vollständigkeit des Systems der Eigenfunktionen und Hauptfunktionen irregulärer Operator-büschel. *Math. Z.*, 1984, vol. 188, pp. 55–68.
8. Tikhomirov S. A. *Finite-dimensional perturbations of Volterra integral operators in the space of vectorfunctions*. Diss. Cand. Sci. (Phys. and Math.). Saratov, 1987. 126 p. (in Russian).
9. Vagabov A. I. *Vvedenie v spektral'nuiu teoriyu differentsial'nykh operatorov* [Introduction to spectral theory of differential operators]. Rostov-on-Don, Izd-vo Rost. un-ta, 1994. 160 p. (in Russian).
10. Rykhlov V. S. On multiple completeness of the root functions of ordinary differential polynomial pencil with constant coefficients. *Proceedings of the Crimean autumn mathematical school-symposium*, CMFD, 2017, vol. 63, no. 2, pp. 340–361 (in Russian). DOI: https://doi.org/10.22363/2413-3639-2017-63-2-340-361
11. Rykhlov V. S. Multiple Completeness of the Root Functions for a Certain Class of Pencils of Ordinary Differential Operators. *Results in Mathematics*, 2017, vol. 72, iss. 1–2, pp. 281–301. DOI: https://doi.org/10.1007/s00025-016-0599-7

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