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Harmonic Analysis of Operator Semigroups Slowly Varying at Infinity

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The article focuses on studying of strongly continuous bounded operator semigroups. In the space of uniformly continuous functions with values in a complex Banach space we consider the subspace of integrally vanishing at infinity functions. This subspace includes the subspace of vanishing at infinity functions, but it is wider. We study the properties of the subspace under consideration. We introduce the definition of slowly varying at infinity (with regard to the subspace of integrally vanishing at infinity functions) function and study the conditions under which a uniformly continuous function belongs to this type. We also introduce the definition of slowly varying at infinity (with regard to the subspace of integrally vanishing at infinity functions) operator semigroup, study its properties and derive the conditions under which a strongly continuous bounded operator semigroup belongs to this type. The results derived in the article might be useful for research of stabilization of parabolic equations solutions with unlimited increase of time.

Keywords: operator semigroup, slowly varying at infinity function, slowly varying at infinity operator semigroup, Beurling spectrum, Banach module.

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