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## The Solvability of a System of Nonlinear Integral Equations of Hammerstein Type on the Whole Line

Kh. A. Khachatryan

Khachatur A. Khachatryan, <https://orcid.org/0000-0002-4835-943X>, Institute of Mathematics of National Academy of Sciences of Armenia, 24/5 Marshal Baghramian Ave., Yerevan 0019, Republic of Armenia, Khach82@rambler.ru

In recent years, the interest has grown in nonlinear integral equations of convolution type in connection with their application in various fields of mathematical physics, in particular, in the  $p$ -adic theory of an open-closed string, kinetic theory of gases, in the theory of radiation transfer in spectral lines. The paper is devoted to the questions of construction of nontrivial solutions and the study of their asymptotic behavior for one system of nonlinear integral equations of convolution type with a symmetric kernel on the whole axis. The results of the work are based on the combination of methods of invariant conical segments construction for the corresponding nonlinear monotone operator with methods of the theory of linear operators of convolution type. A constructive theorem on the existence of two asymptotically different one-parameter families of positive and bounded solutions was formulated and proved, which is the main difference from the previously obtained results. Moreover, from the structure of this system of nonlinear equations follows that all possible shifts of the constructed solutions also satisfy the system. Special attention is paid to the study of the asymptotic behavior of these solutions at the ends of the line. The limits of these solutions in  $\pm\infty$  are calculated and it is proved that the constructed solutions belong to the  $L_1(0, +\infty)$  and  $L_1(-\infty, 0)$  spaces respectively.

*Keywords:* system of equations, vector-function, spectral radius, monotonicity, successive approximations, kernel, Frobenius – Perron theorem.

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