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ОБ УНИВЕРСАЛЬНОСТИ НЕКОТОРЫХ ДЗЕТА-ФУНКЦИЙ

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Хорошо известно, что обобщение дзета функции Гурвица — периодическая дзета функция Гурвица — с трансцендентным параметром универсальна в том смысле, что её сдвигами приближается всякая аналитическая функция. В статье условие трансцендентности параметра заменяется более слабым условием о линейной независимости некоторого множества.

Ключевые слова: периодическая дзета функция Гурвица, пространство аналитических функций, слабая сходимость, универсальность.

1. INTRODUCTION

Let $s = \sigma + it$ be a complex variable, and α, $0 < \alpha \leq 1$, be a fixed parameter. The Hurwitz zeta-function $\zeta(s, \alpha)$ is defined, for $\sigma > 1$, by the Dirichlet series

$$\zeta(s, \alpha) = \sum_{m=0}^{\infty} \frac{1}{(m+\alpha)^s},$$

and continues analytically to the whole complex plane, except for a simple pole at $s = 1$ with residue 1.

A natural generalization of the function $\zeta(s,\alpha)$ is the periodic Hurwitz zeta-function. Let $\mathfrak{a} = \{a_m : m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}\}$ be a periodic sequence of complex numbers with minimal period $k \in \mathbb{N}$. The periodic Hurwitz zeta-function $\zeta(s,\alpha;\mathfrak{a})$ is defined, in the half-plane $\sigma > 1$, by the Dirichlet series

$$\zeta(s,\alpha;\mathfrak{a}) = \sum_{m=0}^{\infty} \frac{a_m}{(m+\alpha)^s}.$$

The periodicity of the sequence $\mathfrak{a}$ shows that, for $\sigma > 1$,

$$\zeta(s,\alpha;\mathfrak{a}) = \frac{1}{k^s} \sum_{l=0}^{k-1} a_l \zeta\left(s, \tfrac{l+\alpha}{k}\right).$$

Thus, the properties of the Hurwitz zeta-function imply the analytic continuation for $\zeta(s,\alpha;\mathfrak{a})$ to the whole complex plane, except for a simple pole at $s = 1$ with residue $a \stackrel{def}{=} \frac{1}{k}\sum\limits_{l=0}^{k-1} a_l$. If $a = 0$, then the function $\zeta(s,\alpha;\mathfrak{a})$ is entire.

Properties of the functions $\zeta(s,\alpha)$ and $\zeta(s,\alpha;\mathfrak{a})$ depend on the parameter α. It is known [6] that the function $\zeta(s,\alpha;\mathfrak{a})$ with transcendental parameter α is universal in the sense that the shifts $\zeta(s+i\tau,\alpha;\mathfrak{a})$, $\tau \in \mathbb{R}$, uniformly on compact subsets of the strip $D = \left\{s \in \mathbb{C} : \frac{1}{2} < \sigma < 1\right\}$, approximate every analytic function. For a precise statement of the universality for $\zeta(s,\alpha;\mathfrak{a})$, we need some notation. Denote by $\mathscr{K}$ the class of compact subsets of D with connected complements. For $K \in \mathscr{K}$, let $H(K)$ denote the class of continuous functions on K which are analytic in the interior of K. Moreover, let $\mathrm{meas}A$ stand for the Lebesgue measure of a measurable set $A \subset \mathbb{R}$. Then the main result of [1] is the following theorem.

Theorem 1. *Suppose that α is a transcendental number, $K \in \mathscr{K}$ and $f(s) \in H(K)$. Then, for every $\varepsilon > 0$,*

$$\liminf_{T\to\infty} \frac{1}{T}\mathrm{meas}\left\{\tau \in [0,T] : \sup_{s\in K} |\zeta(s+i\tau,\alpha;\mathfrak{a}) - f(s)| < \varepsilon\right\} > 0.$$

The aim of the present paper is to replace a hypothesis of Theorem 1 on the transcendence of the parameter α by a wider one. Define the set $L(\alpha) = \{\log(m+\alpha) : m \in \mathbb{N}_0\}$.

Theorem 2. *Suppose that the set $L(\alpha)$ is linearly independent over $\mathbb{Q}$, and that $K \in \mathscr{K}$ and $f(s) \in H(K)$. Then the same assertion as in Theorem 1 is valid.*

Note that if α is a transcendental number, then the set $L(\alpha)$ is linearly independent over $\mathbb{Q}$. On the other hand, it is known [2] that if α is an algebraic irrational number, then at least 51 percent of elements of the set $L(\alpha)$ are linearly independent over $\mathbb{Q}$. Thus, it is possible that the set $L(\alpha)$ is linearly independent over $\mathbb{Q}$ even α is an algebraic irrational number. Unfortunately, we do not know any such α.

For the proof of Theorem 2, a probabilistic method based on limit theorems on the weak convergence of probability measures in the space of analytic functions will be applied.

2. LIMIT THEOREMS

Denote by $H(D)$ the space of analytic functions on D equipped with the topology of uniform convergence on compacta. Let $\mathscr{B}(X)$ stand for the Borel field of the space X. In this section, we consider the weak convergence of the probability measure

$$P_T(A) \stackrel{def}{=} \frac{1}{T}\mathrm{meas}\left\{\tau \in [0,T] : \zeta(s+i\tau,\alpha;\mathfrak{a}) \in A\right\}, \qquad A \in \mathscr{B}(H(D)),$$

as $T \to \infty$.

Let γ be the unit circle on the complex plane, i. e., $\gamma = \{s \in \mathbb{C} : |s| = 1\}$. We start with a limit theorem on the torus $\Omega = \prod\limits_{m=0}^{\infty} \gamma_m$, where $\gamma_m = \gamma$ for all $m \in \mathbb{N}_0$. Since Ω with the product topology and pointwise multiplication is a compact topological Abelian group, on $(\Omega, \mathscr{B}(\Omega))$ the probability Haar measure m_H can be defined, and we have a probability space $(\Omega, \mathscr{B}(\Omega), m_H)$. Let, for $A \in \mathscr{B}(\Omega)$,

$$Q_T(A) \stackrel{def}{=} \frac{1}{T}\text{meas}\left\{\tau \in [0,T] : \left((m+\alpha)^{-i\tau} : m \in \mathbb{N}_0\right) \in A\right\}.$$

Lemma 1. *Suppose that the set $L(\alpha)$ is linearly independent over $\mathbb{Q}$. Then Q_T converges weakly to the Haar measure m_H as $T \to \infty$.*

Proof. Denote by ω the elements of Ω. For $m \in \mathbb{N}_0$, let $\omega(m)$ be the projection of $\omega \in \Omega$ to the coordinate space γ_m. Then it is well known, see, for example, [3], that the Fourier transform $g_T(\underline{k})$, $\underline{k} = (k_1, k_2, \dots)$, of the measure Q_T is of the form

$$g_T(\underline{k}) = \frac{1}{T}\int_0^T \exp\left\{-i\tau \sum_{m=0}^{\infty} k_m \log(m+\alpha)\right\} \mathrm{d}\tau, \qquad (1)$$

where only a finite number of integers k_m are distinct from zero. Now we essentially apply the linear independence of the set $L(\alpha)$. Since $\sum_{m=0}^{\infty} k_m \log(m+\alpha) = 0$ if and only if all $k_m = 0$, we deduce from (1) that

$$\lim_{T\to\infty} g_T(\underline{k}) = \begin{cases} 1 & \text{if} \quad \underline{k} = \underline{0}, \\ 0 & \text{if} \quad \underline{k} \neq \underline{0}. \end{cases}$$

This and Theorem 1.4.2 of [4] show that the measure Q_T converges weakly to m_H as $T \to \infty$. □

Now we will prove a limit theorem for absolutely convergent Dirichlet series. For a fixed $\sigma_1 > 1/2$, and $m \in \mathbb{N}_0$, $n \in \mathbb{N}$, let $v_n(m,\alpha) = \exp\left\{-\left(\frac{m+\alpha}{n+\alpha}\right)^{\sigma_1}\right\}$. Define

$$\zeta_n(s,\alpha;\mathfrak{a}) = \sum_{m=0}^{\infty} \frac{a_m v_n(m,\alpha)}{(m+\alpha)^s}.$$

Then it is known [3] that the latter series is absolutely convergent for $\sigma > 1/2$. For $A \in \mathscr{B}(H(D))$, define $P_{T,n}(A) = \frac{1}{T}\text{meas}\left\{\tau \in [0,T] : \zeta_n(s+i\tau,\alpha;\mathfrak{a}) \in A\right\}$. For $\omega \in \Omega$, define one more function

$$\zeta_n(s,\alpha,\omega;\mathfrak{a}) = \sum_{m=0}^{\infty} \frac{a_m \omega(m) v_n(m,\alpha)}{(m+\alpha)^s},$$

clearly, the series being absolutely convergent for $\sigma > \frac{1}{2}$. Let $\omega_0 \in \Omega$ be a fixed element. On $(H(D), \mathscr{B}(H(D)), m_H)$, define one more probability measure

$$\hat{P}_{T,n}(A) = \frac{1}{T}\text{meas}\left\{\tau \in [0,T] : \zeta_n(s+i\tau,\alpha,\omega_0;\mathfrak{a}) \in A\right\}.$$

Lemma 2. *Suppose that the set $L(\alpha)$ is linearly independent over $\mathbb{Q}$. Then the both measures $P_{T,n}$ and $\hat{P}_{T,n}$ converges weakly to the same probability measure P_n on $(H(D), \mathscr{B}(H(D)))$ as $T \to \infty$.*

Proof. A proof uses Lemma 1, Theorem 5.1 of [5] and the invariance of m_H, and is independent on the arithmetic nature of the parameter α. Therefore, it remains the same as in the case of transcendental α [1]. □

The next step in the investigation of the weak convergence of the measure P_T consists of the approximation of the function $\zeta(s,\alpha;\mathfrak{a})$ by $\zeta_n(s,\alpha;\mathfrak{a})$ in the mean. The space $H(D)$ is metrisable. Denote by ρ a metric in $H(D)$ which induces the topology of uniform convergence on compacta.

Lemma 3. *We have*

$$\lim_{n\to\infty} \limsup_{T\to\infty} \frac{1}{T}\int_0^T \rho(\zeta(s+i\tau,\alpha;\mathfrak{a}), \zeta_n(s+i\tau,\alpha;\mathfrak{a}))\mathrm{d}\tau = 0.$$

Proof. A proof of the lemma in the case of transcendental α in [1] does not use the transcendence property. Therefore, it also remains the same in our case. □

Let, for $\omega \in \Omega$,

$$\zeta(s,\alpha,\omega;\mathfrak{a}) = \sum_{m=0}^{\infty} \frac{a_m\omega(m)}{(m+\alpha)^s}.$$

Then $\zeta(s,\alpha,\omega;\mathfrak{a})$ is the $H(D)$-valued random element defined on the probability space $(\Omega, \mathscr{B}(\Omega), m_H)$ [6].

The approximation of the function $\zeta(s,\alpha,\omega;\mathfrak{a})$ by $\zeta_n(s,\alpha,\omega;\mathfrak{a})$ is more complicated, we need some elements of ergodic theory. For $\tau \in \mathbb{R}$, define $a_\tau = \{((m+\alpha)^{-i\tau} : m \in \mathbb{N}_0)\}$. Let $\{\varphi_\tau : \tau \in \mathbb{R}\}$, where $\varphi_\tau(\omega) = a_\tau\omega$, $\omega \in \Omega$. Then $\{\varphi_\tau : \tau \in \mathbb{R}\}$ is a group of measurable measure preserving transformations of the torus Ω. We will prove the ergodicity of the group $\{\varphi_\tau : \tau \in \mathbb{R}\}$. We recall that the set $A \in \mathscr{B}(\Omega)$ is invariant with respect to the group $\{\varphi_\tau : \tau \in \mathbb{R}\}$ if, for any $\tau \in \mathbb{R}$, the sets A and $A_\tau = \varphi_\tau(A)$ differ one from another by a set of m_H-measure zero. The group $\{\varphi_\tau : \tau \in \mathbb{R}\}$ is invariant if the σ-field of all invariant sets consists of the sets having m_H-measure 0 or 1.

Lemma 4. *Suppose that the set $L(\alpha)$ is linearly independent over $\mathbb{Q}$. Then the group $\{\varphi_\tau : \tau \in \mathbb{R}\}$ is ergodic.*

Proof. It is well known that the characters χ of the group Ω are of the form $\chi(\omega) = \prod_{m=0}^{\infty} \omega^{k_m}(m)$, where only a finite number of integers k_m are distinct from zero. First let χ be a non-trivial character, i. e., $\chi(\omega) \not\equiv 1$. Then we have that

$$\chi(a_\tau) = \prod_{m=0}^{\infty} (m+\alpha)^{-i\tau k_m} = \exp\left\{-i\tau \sum_{m=0}^{\infty} k_m \log(m+\alpha)\right\}. \tag{2}$$

Since the set $L(\alpha)$ is linearly independent over $\mathbb{Q}$, $\sum_{m=0}^{\infty} k_m \log(m+\alpha) \neq 0$ for every finite number of non-zero integers k_m. Therefore, (2) implies that there exists $\tau_0 \in \mathbb{R} \setminus \{0\}$ such that

$$\chi(a_{\tau_0}) \neq 1. \tag{3}$$

Let $A \in \mathscr{B}(\Omega)$ be an invariant set with respect to the group $\{\varphi_\tau : \tau \in \mathbb{R}\}$, and I_A be the indicator function. Then we have that, for every $\tau \in \mathbb{R}$ and for almost all $\omega \in \Omega$,

$$I_A(a_\tau\omega) = I_A(\omega). \tag{4}$$

Denote by $\hat{g}(\chi)$ the Fourier transform of the function g, i. e., $\hat{g}(\chi) = \int_\Omega \chi(\omega)g(\omega)m_H(\mathrm{d}\omega)$. Taking into account (4), we find that

$$\hat{I}_A(\chi) = \chi(a_{\tau_0})\hat{I}_A(\chi).$$

This together with (3) shows that $\hat{I}_A(\chi) = 0$ for every non-trivial character χ.

Now let χ_0 be the trivial character of the torus Ω, i. e., $\chi_0(\omega) \equiv 1$, and let, for brevity, $\hat{I}_A(\chi_0) = b$. Using the orthogonality property of characters and the equality $\hat{I}_A(\chi) = 0$, we find that, for every character χ of the torus Ω,

$$\hat{1}(\chi) = b\int_\Omega \chi(\omega)m_H(\mathrm{d}\omega) = b\hat{I}_A(\chi) = \hat{b}(\chi).$$

From this the lemma easily follows. □

Lemma 5. *Suppose that the set $L(\alpha)$ is linearly independent over $\mathbb{Q}$. Then, for almost all $\omega \in \Omega$,*

$$\lim_{n\to\infty} \limsup_{T\to\infty} \frac{1}{T}\int_0^T \rho\left(\zeta(s+i\tau,\alpha,\omega;\mathfrak{a}), \zeta_n(s+i\tau,\alpha,\omega;\mathfrak{a})\right) \mathrm{d}\tau = 0.$$

Proof. In [6], the assertion of the lemma is proved in the case of transcendental α, however, the transcendence is used only for the proof of the ergodicity of the group $\{\varphi_\tau : \tau \in \mathbb{R}\}$. Since, by Lemma 4, the group $\{\varphi_\tau : \tau \in \mathbb{R}\}$ is ergodic, the proof of the lemma runs in the same way as in [6]. □

Now we are able to obtain the weak convergence of the measure P_T. However, having in mind the identification of the limit measure, we also consider the measure

$$\hat{P}_T(A) = \frac{1}{T}\text{meas}\left\{\tau \in [0,T] : \zeta(s+i\tau,\alpha,\omega;\mathfrak{a}) \in A\right\}, \qquad A \in \mathscr{B}(H(D)).$$

Lemma 6. *Suppose that the set $L(\alpha)$ is linearly independent over $\mathbb{Q}$. Then the both measures P_T and $\hat{P}_T$ converge weakly to the same probability measure P on $(H(D),\mathscr{B}(H(D)))$ as $T \to \infty$.*

Proof. The used method is the same as in the case of transcendental α [1, 6], and uses Lemmas 2, 3 and 5. □

Now we state the main limit theorem of this section.

Theorem 3. *Suppose that the set $L(\alpha)$ is linearly independent over $\mathbb{Q}$. Then the measure P_T converges weakly to the distribution P_ζ of the random element $\zeta(s,\alpha,\omega;\mathfrak{a})$.*

Proof. We apply Lemmas 4, 6 and the Birkhoff–Khinchine theorem. □

3. PROOF OF THE UNIVERSALITY THEOREM

For the proof of Theorem 2, together with Theorem 3 we need the explicit form of the support of the measure P_ζ.

The support of P_ζ is independent of the arithmetic nature of the parameter α, therefore we may use the following result of [1].

Theorem 4. *The support of the measure P_ζ is the whole of $H(D)$.*

Proof of Theorem 2. By the Mergelyan theorem [7], there exists a polynomial $p(s)$ such that

$$\sup_{s\in K}|f(s)-p(s)| < \tfrac{\varepsilon}{2}. \tag{5}$$

In view of Theorem 4, the polynomial $p(s)$ is an element of the support of the measure P_ζ. Thus, for every open neighbourhood G of the polynomial $p(s)$, the inequality $P_\zeta(G) > 0$ is true. Let $G = \{g \in H(D) : \sup_{s\in K}|g(s)-p(s)| < \varepsilon/2\}$. Using Theorem 3, an equivalent of the weak convergence of probability measures in terms of open sets and the definition of G, we obtain that

$$\liminf_{T\to\infty}\frac{1}{T}\text{meas}\left\{\tau \in [0,T] : \sup_{s\in K}|\zeta(s+i\tau,\alpha;\mathfrak{a}) - p(s)| < \varepsilon/2\right\} > 0. \tag{6}$$

It remains to replace in this inequality $p(s)$ by $f(s)$. We note that in view of (5), for such τ,

$$\sup_{s\in K}|\zeta(s+i\tau,\alpha;\mathfrak{a}) - f(s)| < \varepsilon.$$

Thus, we deduce from (6) that

$$\liminf_{T\to\infty}\frac{1}{T}\text{meas}\left\{\tau \in [0,T] : \sup_{s\in K}|\zeta(s+i\tau,\alpha;\mathfrak{a}) - f(s)| < \varepsilon\right\} > 0.$$

The theorem is proved. □

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On Universality of Certain Zeta-functions

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It is well known that a generalization of the Hurwitz zeta-function — the periodic Hurwitz zeta-function with transcendental parameter is universal in the sense that its shifts approximate any analytic function. In the paper, the transcendence condition is replaced by a simpler one on the linear independence of a certain set.

Key words: periodic Hurwitz zeta-function, space of analytic functions, universality, weak convergence.

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К ОЦЕНКЕ ОДНОГО КЛАССА СУММАТОРНЫХ ФУНКЦИЙ

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Для конечнозначных функций натурального аргумента $h(n)$, имеющих ограниченную сумматорную функцию, оцениваются сумматорные функции вида $\sum\limits_{n\le x} h(n)n^{it}$, $1 \le |t| \le T$.

Ключевые слова: числовые характеры, сумматорные функции, степенные ряды.

В работе [1] было показано, что для числовых характеров Дирихле χ при любом действительном t имеет место оценка вида

$$\sum_{n\le x} \chi(n)n^{it} = O(1).$$

В данной работе этот результат обобщается на случай конечнозначных функций натурального аргумента $h(n)$, для которых выполняются условия:

1) $S(x) = \sum\limits_{n\le x} h(n) = O(1)$;

2) функция $g(x)$, заданная степенным рядом вида $g(x) = \sum\limits_{n=1}^{\infty} h(n)x^n$, имеет конечный предел в точке $x = 1$.