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Weighted Integrability of Sums of Series with Respect to Multiplicative Systems

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A necessary and sufficient condition for L^p-integrability with power weight of a function f represented by the series with respect to multiplicative systems with generalized monotone coefficients is obtained. The integrability of the majorant of partial sums of a representing series is also described by the same conditions. In addition we study the integrability of difference quotient $(f(x) - f(0))/x$.

Key words: weighted L^p-integrability, multiplicative system, power weight, generalized monotone sequences.

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