The Problem of Leont'ev on Entire Functions of Completely Regular Growth

V. B. Sherstyukov

National Research Nuclear University MEPhI, Russia, 115409, Moscow, Kashirskoe shosse, 31, shervb73@gmail.com

We consider an entire function of exponential type with all its zeros are simple and form a sequence with the index condensation zero. On the set of zeros a function of its derivative is growing quickly. Required to determine whether original function have complete regularity of growth. This problem, which arose in the theory of representation of analytic functions by exponential series was posed by A. F. Leontiev more than forty years ago and has not yet been solved. In this paper we show that the aforesaid problem a positive solution if the function is «not too small» on a straight line.

Key words: Leont'ev problem, function of completely regular growth, index of condensation.

References

1. Leont'ev A. F. On conditions of expandibility of analytic functions in Dirichlet series. *Math. of the USSR-Izvestiya*, 1972, vol. 6, no. 6, pp. 1265–1277. DOI: 10.1070/IM 1972v006n06ABEH001918.
2. Levin B. Ja. *Distributions of zeros of entire functions*. RI, Providence, Amer. Math. Soc., 1964. [Rus. ed.: Levin B. Ja. *Raspredelenie kornei tselykh funktsii*. Moscow, Gostekhizdat, 1956. 632 p.]
3. Leont'ev A. F. *Riady eksponent* [Exponential series]. Moscow, Nauka, 536 p. (in Russian).
4. Leont'ev A. F. *Tselye funktsii. Riady eksponent* [Entire functions. Exponential series]. Moscow, Nauka, 1983, 175 p. (in Russian).
5. Korobeinik Yu. F. Representing systems. *Russian Math. Surv.*, 1981, vol. 36, no. 1, pp. 75–137. DOI: 10.1070/RM1981v036n01ABEH002542.
6. Abanin A. V. *Slabo dostatochnye mnozhestva i absoliutno predstavliaiushchie sistemy*. Diss. dokt. fiz.-mat. nauk [*Weakly sufficient sets and absolutely representing systems*. Dr. phys. and math. sci. diss.]. Rostov on Don, 1995, 268 p.
7. Bratishchev A. V. A type of lower estimate for entire functions of finite order, and some applications. *Math. of the USSR-Izvestiya*, 1985, vol. 24, no. 3, pp. 415–438. DOI: 10.1070/IM1985v024n03ABEH001243.
8. Korobeinik Yu. F. Maksimal'nye i γ-dostatochnye mnozhestva. Prilozheniia k tselym funktsiiam. II [The maximal and γ-sufficient sets. Applications to entire functions]. *Teoriia funktsii, funktsional'nyi analiz i ikh prilozheniia*. Kharkov, 1991, vol. 55, pp. 23–34 (in Russian).
9. Sherstyukov V. B. On a question about γ-sufficient sets. *Siberian Math. J.*, 2000, vol. 41, no. 4, pp. 778–784. DOI: 10.1007/BF02679704.
10. Sherstyukov V. B. On a problem of Leont'ev and representing systems of exponentials. *Math. Notes*, 2003, vol. 73, no. 2, pp. 286–298. DOI: 10.1023/A:1025068527611.
11. Sherstyukov V. B. Ob odnom podklasse tselykh funktsii vpolne reguliarnogo rosta [On a subclass of entire functions of completely regular growth]. *Kompleksnyi analiz. Teoriia operatorov.*

Matematicheskoe modelirovanie. Vladikavkaz, Publ. VNTs RAN, 2006, pp. 131–138 (in Russian).
12. Sherstyukov V. B. On some criteria for completely regular growth of entire functions of exponential type. *Math. Notes*, 2006, vol. 80, no. 1, pp. 114–126. DOI: 10.1007/s11006-006-0115-6.
13. Bratishchev A. V. On a problem of A. F. Leont'ev. *Sov. Math. Dokl*. 1983, vol. 27, pp. 572–574 (in Russian).
14. Mel'nik Yu. I. O predstavlenii reguliarnykh funktsii riadami tipa riadov Dirikhle [On the representation of regular functions by Dirichlet type series]. *Issledovanie po teorii priblizhenii funktsii i ikh prilozhenii*, Kiev, Naukova Dumka, 1978, pp. 132–141 (in Russian).
15. Mel'nik Yu. I. Ob usloviiakh skhodimosti riadov Dirikhle, predstavliaiushchikh reguliarnye funktsii [Conditions for the convergence of Dirichlet series that represent regular functions]. *Matematicheskii analiz i teoriia veroiatnostei*, Kiev, Naukova Dumka, 1978, pp. 120–123 (in Russian).
16. Mel'nik Yu. I. Ob usloviiakh razlozhimosti reguliarnykh funktsii v riady eksponent [On conditions of expandibility of regular functions in exponential series]. *Vsesoiuz. simpozium po teorii approksimatsii funktsii v kompleksnoi oblasti*, Ufa, 1980, pp. 94 (in Russian).
17. Bratishchev A. V. *Bazisy Kete, tselye funktsii i ikh prilozheniia*. Diss. dokt. fiz.-mat. nauk [*Kothe bases, entire functions and their applications*. Dr. phys. and math. sci. diss.]. Rostov on Don, 1997, 248 p.
18. Ingham A. E. A note on Fourier transforms. *J. London Math. Soc.*, 1934, vol. 9, pp. 29–32.
19. Levinson N. *Gap and density theorems*. New York, Amer. Math. Soc., 1940, 246 p.
20. Sedletskii A. M. *Klassy analiticheskikh preobrazovanii Fur'e i eksponentsial'nye approksimatsii* [Classes of analytic Fourier transforms and exponential approximations]. Moscow, Fizmatlit, 2005, 503 p. (in Russian).
21. Levin B. Ja. Pochti periodicheskie funktsii s ogranichennym spektrom [Almost periodic functions with bounded spectrum]. *Aktual'nye voprosy matematicheskogo analiza*, Rostov on Don, 1978, pp. 112–124 (in Russian).