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On Congruence Lattices of Direct Sums of Strongly Connected Commutative Unary Algebras

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A union of mutually disjoint unary algebras is called their direct sum. A unary algebra is said to be strongly connected if it is generated by its arbitrary element. In the present paper we investigate congruence lattices of the class of all algebras with finitely many operations whose every connected component is strongly connected. We give a necessary and sufficient condition for an algebra from this class to have a distributive congruence lattice (Theorem 1). Besides, all distributive congruence lattices of algebras from the above class are described (Theorem 2).

Key words: commutative unary algebra, strongly connected algebra, congruence lattice of an algebra.

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