Covariant Field Equations and d-tensors of Hyperbolic Thermoelastic Continuum with Fine Microstructure

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A non-linear mathematical model of hyperbolic thermoelastic continuum with fine microstructure is proposed. The model is described in terms of 4-covariant field theoretical formalism. Fine microstructure is represented by d-tensors, playing role of extra field variables. A Lagrangian density for hyperbolic thermoelastic continuum with fine microstructure is given and the corresponding least action principle is formulated. 4-covariant field equations of hyperbolic thermoelasticity are obtained. Constitutive equations of microstructural hyperbolic thermoelasticity are discussed. Virtual microstructural inertia is added to the considered action density. It is also concerned to the thermal inertia. Variational symmetries of the thermoelastic action are used to formulate covariant conservation laws in a plane space–time.

Key words: thermoelasticity, microstructure, field, extra field, action, Lagrangian, covariance, symmetry, conservation law, d-tensor, 4-current, energy–momentum tensor.

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