On a Form of the First Variation of the Action Integral Over a Varied Domain

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Field theories of the continuum mechanics and physics based on the least action principle are considered in a unified framework. Variation of the action integral in the least action principle corresponds variations of physical fields while space – time coordinates are not varied. However notion of the action invariance, theory of variational symmetries of action and conservation laws require a wider variation procedure including variations of the space–time coordinates. A similar situation is concerned to variational problems with strong discontinuities of field variables or other a priori unknown free boundaries which variations are not prohibited from the beginning. A form of the first variation of the action integral corresponding variations of space – time coordinates and field variables under one-parametrical transformations groups is obtained. This form is attributed to 4-dimensional covariant formulations of field theories of the continuum mechanics and physics. The first variation of the action integral over a varied domain is given for problems with constraints. The latter are formulated on unknown free boundaries.

Key words: field, action, least action principle, field equations, transformation group, Lie group, infinitesimal generator, variation, varied domain, constraint.

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