UDC 539.3

Axisymmetric growth of a hollow hyperelastic cylinder

S. A. Lychev, A. V. Mark

Institute for Problems in Mechanics of RAS, 101-1, Vernadskogo ave., 119526, Moscow, Russia, lychevsa@mail.ru, A-V-Mark@yandex.ru

The finite deformations of the growing cylinder fabricated of an incompressible elastic material of Mooney – Rivlin type are under consideration. We assume that the deformations are axisymmetric and constant along the cylinder axis. The discrete and continuous types of growing are studied. The analytical solutions of the corresponding boundary-value problems are derived. The computational examples show the convergence of solutions obtained for the discrete growth to corresponding solutions for continuous growth under the following conditions: the number of discrete plies increases while their thickness decreases such that the final volume of growing solid is fixed.

Key words: additive technologies, growing solids, finite deformations, hyperelasticity, continuous growth, discrete growth.

References

1. Gibson I., Rosen D. W., Stucker B. *Additive Manufacturing Technologies. Rapid Prototyping to Direct Digital Manufacturing*. Springer, 2009. 459 p.

2. Choy K. L. Chemical vapour deposition of coatings. *Progress in Materials Science*, 2003, vol. 48, pp. 57–170.

3. Nastasi M., Mayer J. W. *Ion Implantation and Synthesis of Materials*. Springer, 2006. 263 p.

4. Lychev S. A., Manzhirov A. V. The mathematical theory of growing bodies. Finite deformations. *J. Appl. Math. Mech.*, 2013, vol. 77, no. 4, pp. 421–432.

5. Lychev S. A., Manzhirov A. V. Reference Configurations of Growing Bodies. *Mech. Solids*, 2013, vol. 48, no. 5, pp. 553–560.

6. Lychev S. A. Universal Deformations of Growing Solids. *Mech. Solids*, 2011, vol. 46, no. 6, pp. 863–876.

7. Arutiunian N. Kh., Drozdov A. D., Naumov V. E. *Mekhanica rastushih viazkouprugoplastichnikh tel* [Mechanics of viscous, elastic, plastic bodies]. Moscow, Nauka, 1987, 412 p. (in Russian).

8. Lurie A. I. *Nonlinear theory of elasticity*. Amsterdam, North-Holland, 1990, 617 p. (Rus. ed.: Lurie A. I. Nelininay teoria uprugosti. Moscow, Nauka, 1980, 512 p.)

9. Klarbring A., Olsson T., Stalhand J. Theory of residual stresses with application to an arterial geometry. *Archives of Mechanics*, 2001, vol. 59, pp. 341–364.

10. Epstein M., Maugin G. A. Thermomechanics of volumetric growth in uniform bodies. *Int. J. Plasticity*, 2000, vol. 16, no. 7, pp. 951–978.

11. Yavari A. A geometric theory of growth mechanics. *J. Nonlinear Sci.*, 2010, vol. 20, no. 6, pp. 781–830.

12. Manzhirov A. V., Lychev S. A. The mathematical theory of growing solids: Finite deformations. *Doklady Physics*, 2012, vol. 57, no. 4, pp. 160–163.

13. Noll W. Materially uniform simple bodies with inhomogeneities. *Arch. Rat. Mech. Anal.*, 1967, vol. 27, no. 1, pp. 1–32.

14. Wang C.-C. On the geometric structures of simple bodies, a mathematical foundation for the theory of continuous distributions of dislocations. *Arch. Rat. Mech. Anal.*, 1967, vol. 27, no. 1, pp. 33–94.

15. Epstein M. *The Geometrical Language of Continuum Mechanics*. Cambridge Univ. Press, 2010, 312 p.

16. Maugin G. A. *Material Inhomogeneities in Elasticity*. London, Chapman and Hall, 1993, 294 p.

17. Marsden J. E., Hughes T. J. R. *Mathematical Foundations of Elasticity*. New York, Dover Publ., 1994, 556 p.

18. Postnikov M. M. *Gladkie mnogoobrazia* [Smooth manifolds]. Moscow, Nauka, 1987, 478 p. (in Russian).

19. Choquet-Bruhat Y., Dewitt-Morette C., Dillard-Bleick M. *Analysis, Manifolds and Physics : in 2 pt. Pt. 1. Basics*. Amsterdam, Elsevier, 1982, 660 p.

20. Truesdell C., Noll W. *The Non-Linear Field Theories of Mechanics*. Springer, 2004, 602 p.

21. Manzhirov A. V., Lychev S. A. Residual Stresses in Growing Bodies. *Topical Problems in Solid and Fluid Mechanics*. Delhi : Elite Pub. House, 2011, pp. 66–79.

22. Gurtin M. E., Murdoch A. I. A Continuum Theory of Elastic Material Surfaces. *Arch. Rat. Mech. Anal.*, 1975, vol. 27, pp. 291–323.

23. Fihtengolts G. M. *Kurs differentsalnogo i integralnogo ischislenia* [A course of differential and integral calculus : in 3 vol.]. Moscow, Fizmatlit, 2001, vol. 2, 810 p. (in Russian).

24. Eshelby J. D. The continuum theory of lattice defects. *Solid State Physics*, 1956, vol. 3, pp. 79–144;
Eshelby J. D. The Determination of the Elastic Field of an Ellipsoidal Inclusion, and Related Problems. *Proc. Roy. Soc., ser. A241*, 1957, no. 1226, pp. 376–396;
Eshelby J. D. The elastic field outside an ellipsoidal inclusion. *Proc. Roy. Soc., ser. A252*, 1959, no. 1271, pp. 561–569;
Eshelby J. D., Frank F. C., Nabarro F. R. N. The equilibrium of linear arrays of dislocations. *Phil. Mag.*, 1951, vol. 42, no. 327, pp. 351–364.

25. Polyanin A. D., Manzhirov A. V. *Handbook of Integral Equations*. Boca Raton ; London, Chapman and Hall/CRC Press, 2008, 1544 p.

26. Hartman S. Numerical studies on the identification of the material parameters of Rivlin's hyperelasticity using tension-torsion tests. *Acta Mechanica*, 2001, vol. 148, pp. 129–155.