Integrals of the Loewner Equation with Exponential Driving Function

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We consider the qualitative local behavior of trajectories for the ordinary Loewner differential equation with a driving function which is inverse to the exponential function of an integer power. All the singular points and the corresponding singular solutions are described. It is shown that this driving function generates solutions to the Loewner equation which map conformally a half-plane slit along a smooth curve onto the upper half-plane. The asymptotical correspondence between harmonic measures of two slit sides is derived.

Key words: Loewner equation, harmonic measure, singular solutions, driving function, C^1-curve.

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